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THE INCOMPATIBILITY OF VALUED MONEY AND EQUILIBRIUM POLICY

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In an overlapping generations economy, an equilibrium is defined such that at each date, the young agent determines values for that period's monetary and fiscal instruments. A proposition showing money cannot have value in any such equilibrium is proved.

1. Introduction

Overlapping generations economies provide a framework that is well-suited to the study of fiscal and monetary policy in a dynamic general equilibrium setting. However, because these economies typically permit multiple equilibria, one must confront the question of how to choose among the various equilibria. While Ramsey's (1927) optimal taxation paradigm provides one approach, [1] an alternative means is to assume that private agents, not the government, determine what policies to follow and hence what equilibrium obtains.

To this end, I provide an example of a two-period-lived overlapping generations economy in which at each date the agent in his first period of life solves a policy choice problem that enables him to choose values for that period's monetary and fiscal policy instruments. The old agent, alive at the time that these decisions are made, treats them as parameters. [2] It then follows that a competitive equilibrium that satisfies every young agent's policy choice problem, what I refer to as an equilibrium policy, provides a mechanism for choosing among multiple equilibria.

The main result of this paper is a proposition showing that there cannot be valued money in any equilibrium policy. This result is an example of a monetary folk theorem, namely that in an overlapping generations monetary economy with one good, agents in their first period of life will, if given the opportunity, choose to repudiate any existing currency and issue their own instead.

The remainder of this paper is organized as follows. The next section describes the overlapping generations economy and defines a competitive equilibrium. Section 3 defines each agent's policy choice problem and an equilibrium policy. Section 4 concludes the paper by proving that equilibrium policy and valued money are incompatible and provides a brief discussion of the folk theorem underlying the proposition.

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[1] As Kydland and Prescott (1977) show, certain optimal equilibria may be infeasible due to time inconsistency. For examples of time consistent optimal policies in an overlapping generations economy, see Helpman and Sadka (1979).

[2] In Loewy (1988) I consider the opposite situation, namely at each date the old agent solves that period's policy choice problem and the young agent takes these decisions as given.

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2. Environment and competitive equilibrium

The economy consists of two-period-lived overlapping generations. At each date t (where $t = 1, 2, \ldots$), a single agent, agent t, appears who lives during periods t and $t+1$. Agent t has a strictly positive endowment vector $[w_1, w_2, w_3]$ where w_1 and w_2 are his time t and $t+1$ endowments of the economy's single non-storable consumption good and w_3 is his endowment of time t leisure. [3] Agent t's consumer choice problem is to choose non-negative values for time t and $t+1$ consumption, $c_t(t)$ and $c_t(t+1)$, and a value for time t labor, $x_t(t) \in [0, w_3]$, to maximize this utility, $u[c_t(t), c_t(t+1), w_3 - x_t(t)]$, subject to

$$c_t(t) + c_t(t+1)/R(t) \leq w_1 + w_2/R(t) + [1 - \tau(t)]x_t(t), \tag{1}$$

taking the time t labor income tax rate, $\tau(t) < 1$, and the time t gross real rate of return, $R(t) > 0$, as given. $u(\cdot)$ is assumed to be strictly quasiconcave, strictly increasing and consistent with time t and time $t+1$ consumption being gross substitutes. I represent the solution to this problem by the following set of continuously differentiable functions: $x[\tau(t), R(t)]$, $c_2[\tau(t), R(t)]$ and $q[\tau(t), R(t)]$, agent t's supply of labor, and demands for $t+1$ comsumption and for saving, respectively.

At $t = 1$ there also exists a single agent, agent 0, who is currently completing his life. Agent 0 is endowed with $w_2 > 0$ units of the time 1 consumption good and $M(0) > 0$ units of fiat currency. In order to maximize his time 1 consumption, agent 0 inelastically supplies this currency to agent 1 in exchange for the time 1 good.

Finally, there exists a public sector that is required to raise a known quantity $G \geq 0$ of the consumption good in every period. [4] This is accomplished through a combination of labor income tax and currency issue. Accordingly, I write the time t government budget constraint as

$$\tau(t)x_t(t) + p(t)[M(t) - M(t-1)] = G, \qquad t \geq 1, \tag{2}$$

where $p(t)$ is the price of fiat currency in terms of time t good and $M(t)$ is the time t aggregate stock of fiat currency.

Since fiat currency is the economy's only asset, a perfect foresight competitive equilibrium (CE) for this economy is defined as follows:

Definition 1. Given $G \geq 0$ and $M(0) > 0$, a CE consists of a pair of positive sequences $\{1 - \tau(t), R(t)\}_{t=1}^{\infty}$ and a $p(1) \geq 0$ such that

(a) $\tau(1)x[\tau(1), R(1)] + q[\tau(1), R(1)] - p(1)M(0) = G$,

(b) $\tau(t)x[\tau(t), R(t)] + q[\tau(t), R(t)] - R(t-1)q[\tau(t-1), R(t-1)] = G, \quad t \geq 2$,

(c) $q[\tau(t), R(t)] \geq 0, \quad t \geq 1$.

[3] Since agent t is retired during time $t+1$, I suppress the time $t+1$ leisure component in both the endowment vector and the utility function (see below).

[4] Since public sector expenditure is a constant, I omit it as an argument in agents' utility functions.

3. Equilibrium policy

To define an equilibrium policy (EP), it is first necessary to describe agent t's policy choice problem. At each date $t \geq 1$, agent t chooses the time t values of the economy's fiscal and monetary instruments, the time t labor income tax rate and rate of return, respectively, [5] to maximize his indirect utility $v[\tau(t), R(t)]$. In addition, since all other periods' policy instruments are determined in a like fashion, it must be the case that agent t takes the instrument choices of his predecessors, summarized by the time t state variable $M(t-1)$, and of his successors, $\{\tau(j), R(j)\}_{j>t}$, as given.

The need to raise revenue for the government implies that agent t's choices for $\tau(t)$ and $R(t)$ must satisfy the time t government budget constraint. Furthermore, since agent t makes his choices at time t, but the actual value of $R(t)$ depends on $p(t+1)$, and hence on agent $t+1$'s choices at time $t+1$, agent t will limit his choices to those that he perceives to be consistent with his expectations of agent $t+1$'s choices. In other words, given the value of $p(t+1)$ implied by agent t's expectations of $\tau(t+1)$ and $R(t+1)$, $p(t+1)^e$ say, agent t will constrain his choices of $\tau(t)$ and $R(t)$ to those that yield a value for $p(t)$, $p(t)'$ say, such that $p(t+1)^e/p(t)' = R(t)$ if $P(t+1)^e > 0$, or such that $p(t)' = 0$ if $p(t+1)^e = 0$. In order to guarantee this consistency, agent t's choices must satisfy a constraint that links both his choices and his expectations to $p(t+1)$. The time $t+1$ government budget constraint provides the needed link.

It is useful to summarize the previous discussion by providing a complete statement of agent t's policy choice problem: given $M(t-1)$ and $\{\tau(j), R(j)\}_{j>t}$, choose $\tau(t)$ and $R(t)$ to maximize $v[\tau(t), R(t)]$ subject to satisfying the time t and time $t+1$ government budget constraints. With the definition of the policy choice problem now complete, the definition of an EP is immediate.

Definition 2. An EP is a CE that for all dates $t \geq 1$ solves agent t's policy choice problem.

4. The incompatibility of EP and monetary equilibria

In Loewy (1988, Lemma 3.1) I show that the set of stationary CE for the economy of section 2 is compact. From this result I can prove that for a wide class of models, the set of stationary EP is non-empty; it includes at least one non-monetary equilibrium. Rather than provide a proof of this result here, I consider the more interesting question of whether there exist any EP, stationary or otherwise, where money is valued. The following proposition shows that the answer is no.

Proposition 1. No EP can be a monetary equilibrium.

Proof. Let $b^* = \{\tau^*(t), R^*(t)\}_{t=1}^{\infty}$ be an EP and assume, for the purpose of deriving a contradiction, that b^* is a monetary equilibrium. To prove the proposition, it suffices to show that $(\tau^*(1), R^*(1))$ cannot be a solution to agent 1's policy choice problem.

Let $r(\tau, R) = \tau x(\tau, R) + q(\tau, R)$. Since agent 1 is assumed to be on his demand curves, his policy choice problem is to choose $\tau(1)$ and $R(1)$ to maximize $v[\tau(1), R(1)]$ subject to $r[\tau(1), R(1)] - p(1)M(0) = G$ and $r[\tau^*(2), R^*(2)] - R(1)q[\tau(1), R(1)] = G$ taking $M(0)$, $\tau^*(2)$ and $R^*(2)$ as given. However, a CE, and hence an EP, only require that $p(1) \geq 0$ and, given $\tau(1)$ and $R(1)$, market clearing implies that $p(1)$ adjusts to assure that the $t=1$ government budget constraint is satisfied. Therefore, requiring $\tau(1)$ and $R(1)$ to satisfy $r[\tau(1), R(1)] \geq G$ fully describes the conditions that the $t=1$ government budget constraint imposes on agent 1's policy choice problem.

[5] One can interpret using the time t rate of return as the monetary instrument as employing an 'interest rate target'.

Forming a Lagrangian for this problem and taking first-order conditions shows that $\tau^*(1)$ and $R^*(1)$ necessarily satisfy

$$\partial v/\partial\tau + \lambda\,\partial r/\partial r - \mu\,\partial Rq/\partial\tau \le 0; \quad = 0 \quad \text{if} \quad x[\tau^*(1),\ R^*(1)] > 0, \tag{3a}$$

$$\partial v/\partial R + \lambda\,\partial r/\partial R - \mu\,\partial Rq/\partial R = 0, \tag{3b}$$

where λ and μ are the multipliers for $r[\tau(1),\ R(1)] \ge G$ and the $t = 2$ government budget constraint, respectively, and (3b) holds with equality because b^* is a monetary equilibrium implying that $q[\tau^*(1),\ R^*(1)] > 0$.

Consider first the case where (3a) holds with equality. Appealing again to the contrapositive, it follows that $p^*(1) > 0$ which in turn implies that $r[\tau^*(1),\ R^*(1)] > G$, and hence that $\lambda = 0$. Next, note that $c_2(\cdot) = Rq(\cdot) + w_2$ holds identically so that $\partial c_2/\partial i = \partial Rq/\partial i$ for $i = \tau$, R, and that this identity and the $t = 2$ government budget constraint jointly imply that $\mathrm{d}G = -\,\mathrm{d}c_2$. As the envelope theorem and $\lambda = 0$ yield $\partial v/\partial G = -\mu$, it follows that $\mu = \partial u/\partial c_2$. After making these substitutions, writing $\partial v/\partial\tau$ and $\partial v/\partial R$ in terms of the direct utility function and demands, and simplifying, eqs. (3a and 3b) become

$$(\partial u/\partial c_1)(\partial c_1/\partial\tau) = -(\partial u/\partial x)(\partial x/\partial\tau), \tag{4a}$$

$$(\partial u/\partial c_1)(\partial c_1/\partial R) = -(\partial u/\partial x)(\partial x/\partial R). \tag{4b}$$

Finally, imposing the first-order condition $(\partial u/\partial c_1)[1 - \tau^*(1)] = -\partial u/\partial x$ in eqs. (4a and 4b) yields

$$\partial c_1/\partial\tau = [1 - \tau^*(1)](\partial x/\partial\tau), \tag{5a}$$

$$\partial c_1/\partial R = [1 - \tau^*(1)](\partial x/\partial R). \tag{5b}$$

As $c_1(\cdot) + q(\cdot) = [1 - \tau(1)]x(\cdot) + w_1$ also holds identically, eqs. (5a) and (5b) imply that $\tau^*(1)$ and $R^*(1)$ satisfy $\partial q/\partial\tau = -x(\cdot)$ and $\partial q/\partial R = 0$. However, the second equality contradicts an implication of gross substitutes, namely that $\partial q/\partial R > 0$ when $q(\cdot) > 0$. Thus, $(\tau^*(1),\ R^*(1))$ cannot be a solution to agent t's policy choice problem.

Should $x(\tau^*(1),\ R^*(1)] = 0$, then the above argument continues to go through except that the derivative with respect to the tax rate is now interpreted as a left hand derivative and the first-order condition for the tax rate, eqs. (3a)–(5a), now holds with the usual weak inequality. ■

Hendricks, Judd and Kovenock (1980) and Esteban (1982) provide some insight into the folk theorem that underlies this proposition. In two-period-lived overlapping generations economies, the only monetary equilibria in the core are those in which money facilitates intragenerational trade. Thus, if money is only used for intergenerational trade, as it is here, then monetary equilibria cannot be in the core. The difficulty is that if we let all agents $j \ge t$ be a coalition, then this coalition will block any monetary equilibrium since agent t can do better by refusing all existing cuurency and issuing his own instead. [6]

[6] See, in particular, Esteban's (1982) Corollary 1 and his concluding remarks.

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