

# Patent Damages and Spatial Competition

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## Abstract

This paper examines the impact, on market competition and efficient bargaining over a license, of three regimes used to calculate damages in patent cases. We focus on product patents in a differentiated, duopoly setting. We find that regardless of the damage regime, the per-unit royalty under efficient bargaining is not unique. As a result, the “reasonable royalty” damage regime’s application of a “hypothetical negotiation” gives the court significant discretion in assigning damages. However, the “lost profits” regime will typically yield the best incentives to innovate for patents covering the most valuable products, and is the only regime that may deter infringement. The “unjust enrichment” regime is the weakest of the three. Our results offer an efficiency argument for abandoning it.

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## 1. Introduction

The means by which damages are assessed if a patent is found valid and infringed have a substantial effect on how potential litigants behave in the market, on how licenses are contracted, and on firms' payoffs. Since product innovations are particularly difficult to protect with trade secrets, patent protection is quite important for reaping returns to innovation in such cases. Given this, and the fact that the patent grant is fundamentally spatial, it is surprising that the patent literature has ignored, thus far, the effect of the choice of damage regime on spatial competition in the presence of product patents.<sup>1</sup> Perhaps most importantly, the literature has yet to model bargaining with richness sufficient to characterize the potential gains to efficient licensing of a second firm, which are key to the relative performance of damage regimes. We seek to fill these gaps.

Using a Hotelling (1929) model of duopoly competition, we analyze the impact of damage regimes on Coasian bargaining between a patentee and an imitator over a license and on their payoffs. Efficient bargaining obtains in equilibrium regardless of damage regime, so total static welfare is maximized in all cases. However, the payoffs earned by the patentee and imitator differ across regimes, because the threat points depend on how damages would be calculated were bargaining to break down and infringing market competition ensued. Indeed, the net gains to being the patentee instead of the imitator, which we interpret as *incentives to innovate*, depend crucially upon which damage regime is chosen.

We consider the three primary damage regimes that have been used by United States courts: "reasonable royalty," where damages are based on a hypothetical (pre-infringement) bargain between patentee and imitator, "lost profits," where damages restore the patentee to a hypothetical monopoly profit, and "unjust enrichment," where the imitator must disgorge all profit.<sup>2</sup> We show that the reasonable royalty regime gives the court significant discretion in assigning damages, because the hypothetical bargain approach does not pin down unique

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<sup>1</sup>Only two papers consider the effect of damage regime on product patents (Choi 2006 and Heath et al. 2002), and both consider models of quantity competition, with no deviation in the products' demands.

<sup>2</sup>Unlike other authors, we don't consider "injunctions" as a damage regime, because our model is a one-shot game. In other words, if competition is taking place, there is no injunction. If an injunction is imposed, it just forces the bargaining equilibrium, and therefore, doesn't change our analysis. Heath et al. (2002) discuss damage rules for other countries.

fixed or per-unit components. If the court chooses these components to maximize incentives to innovate, then the reasonable royalty regime generates the highest incentives to innovate if patent enforcement is certain. Intuitively, this regime maximizes static welfare under threat-point competition, generating the highest possible total profit to divide between patentee and imitator. On the other hand, if patent enforcement is less than certain, then the lost profits regime generates the highest incentives to innovate for sufficiently valuable products. Lost profits also has the advantage that it is the only damage regime that may preclude infringement outside of bargaining. The unjust enrichment regime is the weakest of the three, as it never generates the highest incentives to innovate.<sup>3</sup> Our findings suggest that the Patent Act of 1946 and the 1964 Supreme Court decision in *Aro Manufacturing Co. v. Convertible Top Replacement Co.*,<sup>5</sup> which together ended the use of unjust enrichment damages, are supportable on economic grounds.

Our model enriches the analysis of damages for several reasons. Because of location economies, it is both privately and socially optimal for the patentee to license a second firm, so a non-trivial licensing contract obtains in equilibrium.<sup>6</sup> Additionally, a contract implements the joint-profit-maximizing prices only for particular per-unit royalties. This setting contrasts with the case of Cournot competition with constant marginal costs (e.g. Choi 2006; Anton and Yao 2007), where joint profit is maximized under monopoly, and the case of bilateral monopoly licensing of research tools (e.g. Schankerman and Scotchmer 2001), where only fixed-fee licensing is considered.<sup>7</sup>

The most natural interpretation of our model is pure spatial competition, e.g. the patentee is a company with production facilities in New York, and the potential imitator is located in California. In this case, the transportation costs are interpreted as the cost of shipping

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<sup>3</sup>Only the first two regimes are still in use today. Courts decide which to apply case by case, and the methods are the subject of much discussion in the legal literature.<sup>4</sup>

<sup>5</sup>377 US 476.

<sup>6</sup>Heath et al. (2002) consider a model of capacity-constrained firms engaging in competition under various damage regimes, which also allows duopoly profits to be greater than monopoly profits. The mechanism is distinct, however.

<sup>7</sup>Using a different approach, Ayres and Klemperer (1999) argue that making patent enforcement probabilistic and delaying damage awards is superior to injunctive relief. The intuition is that, by granting the patentee lower per-period expected profit while lengthening the patent term, the patentee's incentives to innovate can be held constant. At the same time, the reduction in deadweight losses, due to lower patentee effective market power, increase total welfare.

the product. Alternatively, the model could be interpreted as reflecting product differentiation, where the transportation costs represent the consumer’s loss of utility of not having the “ideal” product. In either case, location economies influence bargaining and equilibrium pricing.

In addition to yielding a richer context for bargaining, our framework has significant practical appeal. First, the patent grant is fundamentally, and multi-dimensionally, spatial. The patent issuing country’s boundaries determine the covered geographic area, while the patent’s claims determine the covered product characteristics.<sup>8</sup> Studying licensing in this context is therefore important to understanding patent licensing generally. Second, the strategic complementarity of prices has yet to be analyzed by this literature. We show that complementarity is enhanced by both royalties and the likelihood of patent enforcement, so prices may actually be higher under infringement. This may have some implications for techniques used to calculate damages, which typically assume that infringement causes prices to fall. Finally, product patents are highly represented in litigation, and the widespread application of the doctrine of equivalents implies significant differentiation.<sup>9</sup> Perhaps most notably, each of the three pivotal decisions on damage awards cited in this introduction involves product patents only,<sup>10</sup> and the test for whether lost profits apply (described below) seems particularly geared toward them.

Prior to *Aro*, courts usually awarded damages on the basis of the imitator’s profit. In *Aro*, the Court found that when Congress amended the statute in 1946, it intended to proscribe unjust enrichment damages.<sup>11</sup> It is clear from the statutory language that the preferred measure of compensation is lost profits. In the landmark decision in *Panduit Corp. v. Stahlin Bros. Fibre Works, Inc.* (1978),<sup>12</sup> the court also announced four factors a patentee

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<sup>8</sup>Merges and Nelson (1990, p. 839) state, “The economic significance of a patent depends on its scope.” The spatial nature of patents has also been analyzed by Klemperer (1990), who studies the welfare tradeoffs between patent scope and length.

<sup>9</sup>See Merges and Nelson (1990).

<sup>10</sup>In *Aro*, the patent at issue, No. 2,569,724 “Convertible Folding Top with Automatic Seal at Rear Quarter,” is a mechanism for convertible automobiles. In *Georgia-Pacific Corporation v. U.S. Plywood-Champion Papers Inc.* (166 USPQ 235 [S.D.N.Y]), the patent at issue, No. 2,286,068 “Plywood Panel,” is a plywood product. In *Panduit Corp. v. Stahlin Bros. Fibre Works, Inc.* (197 USPQ 726 [6 Cir]), the patent at issue, No. 3,024,301 “Wiring Grille,” is a type of duct for wiring of electrical control systems.

<sup>11</sup>The amended statute (35 USC Section 284) states that a patentee is entitled to “damages adequate to compensate for the infringement, but in no event less than a reasonable royalty.”

<sup>12</sup>197 USPQ 726 [6 Cir].

must prove to recover lost profits:

- (1) demand for the patented product,
- (2) absence of acceptable noninfringing substitutes,
- (3) his manufacturing and marketing capability to exploit the demand, and
- (4) the amount of the profit he would have made.

When the *Panduit* test cannot be satisfied,<sup>13</sup> many courts resort to awarding a reasonable royalty on the basis of a hypothetical (pre-infringement) arm’s-length negotiation between the patentee and the imitator. In *Georgia-Pacific Corporation v. U.S. Plywood-Champion Papers Inc.* (1970),<sup>14</sup> the court laid out a fifteen-factor test for determining the royalty rate, which has been widely followed by other courts. Coolley (1993) shows that courts awarded reasonable royalty damages more often than lost profits during 1982-92, but the application of lost profits typically resulted in higher damages.

We attempt to model damage regimes to most closely resemble the letter of the law.<sup>15</sup> In modeling the reasonable royalty regime, we base the fixed and per-unit royalty components on a hypothetical negotiation between the patentee and the infringer, where such negotiation assumes that the patent is valid and would be infringed absent the license. This is consistent with the way Schankerman and Scotchmer (2001) model “lost royalties.” In modeling the lost profits regime, we base damages on the patentee’s hypothetical payoffs as a monopolist (i.e., absent infringement), not on his hypothetical payoff under bargaining. Under the unjust enrichment regime, the court bases damages only on the actual profit earned by the imitator. This is consistent with the way Anton and Yao (2007) model these damage regimes.<sup>16</sup>

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<sup>13</sup>It may not even be possible to satisfy (1) when the patent is for a research tool or a process. We know of no case where “lost profits” has been applied for either of those types of patents.

<sup>14</sup>166 USPQ 235 [S.D.N.Y].

<sup>15</sup>Unlike Schankerman and Scotchmer (2001), we interpret “lost profits” to not include lost royalty income, but only direct loss of sales.

<sup>16</sup>We are sympathetic to the approach of Schankerman and Scotchmer (2001, 2005) of basing lost profit damages entirely on “equilibrium” profits achieved through bargaining. This approach may be appropriate when court enforcement is certain, as in the Schankerman and Scotchmer (2001) setting, and it is the approach we take with respect to the “hypothetical bargain” in the reasonable royalty regime. However, mimicking their approach would lead us to the unsavory proposition that damages might depend on ex ante beliefs about court enforcement. In section 7 of their 2001 paper, for example, they identify unique damages under the assumption that bargaining leads to efficiency gains. However, this level of damages is based on their finding that infringement is always deterred if bargaining breaks down. This holds only when court

We show that the “hypothetical bargain” approach to determining reasonable royalty damages does not pin down unique royalties, leaving the court significant discretion. As in the setting of Schankerman and Scotchmer (2001), the court has a continuum of possible fixed fees it can award because of a circularity—the choice of the “reasonable” fixed fee determines the equilibrium fixed fee under the hypothetical bargain. If it is willing to assume parties expect particular threat-point equilibria, we show that the court also has a continuum of possible per-unit royalties it can award. The mechanism driving this latter result, multiple equilibria, is distinct.

The way the court exercises that discretion matters greatly for the extra equilibrium profit earned by the patentee, which we interpret as the incentives to innovate. By choosing the largest possible fixed fee consistent with the “hypothetical bargain,” it maximizes incentives to innovate. Basing “reasonable” fixed fees on *observed* bargains is quite likely to cause the patentee to earn less profit and to reduce incentives to innovate, however. When patent enforcement is uncertain, we find that the patentee and imitator bargain to an equilibrium fixed fee that is strictly lower than the “reasonable” fixed fee that the parties expect the court would impose in damages, whenever the latter is positive.

Hence, we find support for the reasoning about the drawbacks of using standard royalties, made by the *Panduit* court:

“Except for the limited risk that the patent owner, over years of litigation, might meet the heavy burden of proving the four elements required for recovery of lost profits, the infringer would have nothing to lose, and everything to gain if he could count on paying only the normal, routine royalty non-infringers might have paid...**(T)he infringer would be in ‘heads-I-win, tails-you-lose’ position.**” (197 USPQ 726 [6 Cir], emphasis ours.)

If the court instead uses the incentive-maximizing fixed fee and patent enforcement is certain, 

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enforcement of the patent is certain. If this is relaxed, then infringement is not necessarily deterred, and when it is not, the damage award depends directly on the likelihood of court enforcement. This result, which obtains in the model of the 2005 paper, is awkward, because after a patent is found valid and infringed, the likelihood of enforcement is unity and there is no provision in the law for basing damages on prior beliefs about the outcome of litigation. Importantly, there is no provision in the *Georgia-Pacific* factors for such beliefs.

then the reasonable royalty regime generates the highest incentives to innovate, regardless of product value. The main strength of the reasonable royalty regime is that it yields symmetric prices under threat-point competition, maximizing static welfare. When the likelihood of patent enforcement is very high, the firms would implement near-collusive prices. When the maximum fixed fee is chosen, nearly all of the imitator's expected profit is transferred to the patentee. Hence, the patentee earns nearly all of the collusive profit.

As the likelihood of patent enforcement falls, the patentee's threat-point profit under the reasonable royalty regime falls, because the firms' prices would fall. For less-than-perfect enforcement, the lost profits regime generates the highest incentives to innovate for sufficiently valuable products. Its main strength is that it is the only regime that may deter infringement under threat-point competition; when it does so, it yields incentives to innovate that equal monopoly profit. This profit increases with product value at rate one. On the other hand, the incentives to innovate under the reasonable royalty regime increase with product value only at the rate of the likelihood of patent enforcement. Hence, for sufficiently high product value, the lost profits regime generates higher incentives.

Schankerman and Scotchmer (2001) study the impact of the "lost royalties," unjust enrichment, and injunction regimes under certain court enforcement. For their main case, research tools to be used by another party, there is no meaningful distinction between lost profits and lost royalties. In addition to their findings on the circularity of reasonable fixed fees, they find that infringement is not necessarily deterred, and that the patentee might prefer an enforcement regime that leads to infringement, absent a license.

Anton and Yao (2007) restrict attention to process patents, and examine the lost profits, reasonable royalties, and disgorgement (unjust enrichment) regimes in a Cournot duopoly model. They show that, under the lost profits regime, the imitator can infringe without diminishing the patentee's profit, and that this "passive" infringement is an equilibrium for high levels of court enforcement of patents.<sup>17</sup> In contrast, we find that when bargaining breaks down, passive infringement may be an equilibrium, but it is not generally unique.

Anton and Yao also find that the other party is always better off infringing (either

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<sup>17</sup>Basing lost profits on equilibrium licensing (see footnote 13), Schankerman and Scotchmer (2005) show that dissipation of total profits under infringement is a crucial determinant of infringement deterrence.

passively or actively) than not entering the market. Consequently, the penalty for losing a patent race is lower and the incentive to innovate is blunted. Because passive infringement is not always possible in our setting, we do not find the same effect. Indeed, the lost profits regime is the only one that may deter infringement, and it produces strong incentives to innovate in such situations.

Choi (2006) studies product patents in a Cournot duopoly model. He finds that the lost profits regime generates greater profit for the patentee and greater R&D incentives. This result does not hinge on whether infringement is deterred by these regimes. Choi also finds that if patent enforcement is uncertain, then no “reasonable” royalty rate exists when the patentee makes a take-it-or-leave-it offer in bargaining over a license. This finding emerges because market profit is maximized under monopoly in a Cournot model, and suggests, indirectly, that efficient bargaining should eliminate competition. It does not extend to Coasian bargaining over a license prior to competition in a differentiated products setting, as we show. Indeed, we find a multiplicity of possible “reasonable” royalty rates, regardless of the strength of patent enforcement.

The only paper we are aware of that studies patent licensing under spatial competition is Poddar and Sinha (2004). They too consider “inside” licensing (i.e. the licensor is a producer) but restrict attention to the case of a process patent and do not study uncertain enforcement or damage regimes. Indeed, their work is done in the spirit of the literature on optimal patent licensing — their main results compare the various ways of selling a license in a non-cooperative framework.<sup>18</sup> Our work, though related, does not take issue with any of the findings in this literature.

## 2. The Model

We begin with a standard model of the Hotelling linear city with fixed firm locations.<sup>19</sup> Consumers have identical reservation value  $V$  for the good and are distributed uniformly

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<sup>18</sup>This large literature has considered fixed fees, per-unit royalties, auctions, and combinations of fixed fees and royalties, for “outside” licensing of process patents (Kamien and Tauman 1986) and product patents (Katz and Shapiro 1986), as well as for “inside” licensing (Marjit 1990). See Kamien (1992) for a survey.

<sup>19</sup>The assumption of fixed locations implies that the firms have sunk costs, e.g. plant location.

along a line of unit length. They bear transportation costs  $t$  per unit of distance they travel to the seller. The patentee sells at location 0. There are two possible market structures: (1) single-price monopoly, and (2) duopoly including an imitator, who sells at location 1. All parties are risk neutral and their preferences do not display wealth effects.

Before market activity, the patentee and potential imitator bargain over a licensing contract that may specify a per-unit royalty  $r$ , paid from the imitator to the patentee, and a fixed payment  $F$ . The parties may also fix prices and/or market shares, but any contract must be *self-reinforcing*, i.e., conditional on the specified royalties, neither party has an incentive to deviate from the specified prices or market shares.<sup>20</sup> If they fail to agree, market activity ensues in the shadow of an exogenously specified damage regime that is common knowledge.

If the imitator competes, the firms reach a Nash equilibrium in prices. We denote the patentee's price as  $P_H$ ,<sup>21</sup> and the imitator's price as  $P_I$ . Demands are  $D_H(P_H, P_I)$  and  $D_I(P_H, P_I)$  and revenues are  $R_H(P_H, P_I) = P_H D_H(P_H, P_I)$  and  $R_I(P_H, P_I) = P_I D_I(P_H, P_I)$ , respectively. The imitator produces with the same constant-marginal-cost technology as the patentee, and marginal costs are normalized to zero. If the imitator does not compete, it earns a payoff of 0.

A consumer located at  $x$  buys from one of the two sellers if  $\text{Max}\{V - P_H - tx, V - P_I - t(1 - x)\} \geq 0$ , and buys from the patentee if  $V - P_H - tx \geq V - P_I - t(1 - x)$  also holds. Demands are determined by prices through these two conditions. The *pivotal* buyer, who is indifferent between buying from the patentee and the imitator, is

$$\hat{x}(P_H, P_I) = \frac{1}{2} + \frac{P_I - P_H}{2t} \in [0, 1]$$

Combining these conditions, one can easily see that the entire market is covered if and only if

$$P_H + P_I \leq 2V - t, \tag{1}$$

in which case  $D_H(P_H, P_I) = \hat{x}(P_H, P_I)$  and  $D_I(P_I, P_H) = 1 - \hat{x}(P_H, P_I)$ .

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<sup>20</sup>Implicitly, we assume that to enforce a contract that is not self-reinforcing would require costly monitoring, making such contracts sub-optimal.

<sup>21</sup>H is for patent *holder*.

For ease of explanation, we restrict attention to the case  $V \geq \frac{3}{2}t$ . This is sufficient to guarantee that the market is fully covered in equilibrium under duopoly and to rule out uninteresting cases where a negative per-unit royalty may be optimal. It also reduces the incidence of multiple equilibria.<sup>22</sup>

The patent covers a brand-new product. In absence of a licensing contract, if the imitator makes positive sales, the patentee takes it to court, and is entitled to damages if the patent is found valid in court. Following Anton and Yao (2007), we assume that the patent is found valid with exogenous, commonly-known probability  $\gamma \in [0, 1]$ ,<sup>23</sup> abstract from transactions costs in the litigation stage,<sup>24</sup> and refer to *passive infringement* as a situation where the imitator's infringing market activity does not result in lost profits for the patentee.

We study three damage regimes. Under the *reasonable royalty* regime, the imitator pays the patentee a per-unit royalty for each unit it sells, as well as a fixed fee. The court chooses these measures to mimic a licensing contract under hypothetical arm's-length bargaining. Under the *lost profits* regime, the imitator pays the patentee the difference between its hypothetical monopoly profit and its actual duopoly profit. Under the *unjust enrichment* regime, the imitator pays the patentee all of its market profits.

The timing of the model is as follows. First, the damage regime is exogenously determined. Second, the patentee and imitator bargain over a license for the patent. Third, the imitator decides whether to compete. If the imitator stays out, the patentee operates as a single-price monopolist. If the imitator enters, then the two firms compete in prices and reach a Nash equilibrium. Finally, if bargaining breaks down and the imitator makes sales, then the court decides the patent's validity, and assigns damages.

## 2.1. Bargaining

We denote the payoffs for the patentee and imitator from their market activity (includ-

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<sup>22</sup>In the standard Hotelling duopoly model with fixed locations, the restriction  $V \geq \frac{3}{2}t$  is the minimum sufficient one to rule out multiple equilibria (see, e.g. Mas-Colell et al. 1995, p. 398 and exercise 12.C.14, p. 432).

<sup>23</sup>An interesting extension of this model would allow the parties' assessments of this probability to differ, thereby allowing for the special case where infringement is inadvertent.

<sup>24</sup>For detailed models of patent litigation, see Meurer (1989), Choi (1998) or Crampes and Langinier (2002).

ing royalty revenue) under an efficient bargain as  $\Pi_H^{B_j}$  and  $\Pi_I^{B_j}$ , and the expected payoffs from market activity plus damages, when bargaining breaks down, as  $\pi_H^j$  and  $\pi_I^j$ , where  $j \in \{RR, LP, UR\}$  depending on the damage regime. These latter payoffs form the *threat points* under bargaining. Following Schankerman and Scotchmer (2001), we abstract from transactions costs of bargaining.

The Coase Theorem implies that the parties will agree to prices that maximize total profit,

$$\Pi_H^{B_j} + \Pi_I^{B_j} = R_H(P_H, P_I) + R_I(P_H, P_I),$$

and that they will implement that profit with a per-unit royalty that renders the contracted prices self-reinforcing. Consider prices first. If the pivotal buyer is not held to reservation utility, then the patentee and imitator can both raise prices by the same amount, maintain market shares, and increase joint profit. Hence, joint profit is maximized only if this buyer earns zero utility, i.e., if (1) holds with equality. Factoring in this condition, and doing a bit of algebra, the firms' problem is

$$\begin{aligned} \text{Max}_{\{P_H, P_I\}} \quad & (V - \frac{1}{2}t) - \frac{1}{2t}(P_I - P_H)^2 \\ \text{s.t.} \quad & P_H + P_I = 2V - t. \end{aligned} \tag{2}$$

Clearly, joint profit is maximized if and only if the efficient bargain results in  $P_H = P_I = V - \frac{1}{2}t$ .

The firms must also choose a per-unit royalty such that neither wishes to deviate from the efficient prices. A well-chosen positive  $r$  can achieve this because it increases the cost of gaining market share, enhancing the strategic complementarity of prices. For the imitator,  $r$  is an artificial marginal cost of additional market share. For the patentee,  $r$  is an artificial opportunity cost of additional market share. The following result shows that a continuum of per-unit royalties achieve a self-reinforcing, efficient contract.

**Lemma 1** *Regardless of the damage regime, the bargain is efficient only if it yields  $P_H = P_I = V - \frac{1}{2}t$ . This is a self-reinforcing equilibrium if and only if the per-unit royalty  $r \in [V - \frac{3}{2}t, V - t] \equiv \mathcal{R}^*$ .*

For sufficiently low  $r < V - \frac{3}{2}t$ , both firms have the incentive to cut price below  $V - \frac{1}{2}t$  to

increase market share. For sufficiently high  $r > V - t$ , the imitator prefers to reduce royalty payments by charging a higher price than  $V - \frac{1}{2}t$  and earning a smaller market share than the patentee.<sup>25</sup>

Hence, factoring in the per-unit royalty payment of  $\frac{1}{2}r$  and the fixed payment  $F$  from the imitator to the patentee, we can write the equilibrium payoffs for  $r \in \mathcal{R}^*$  and a given  $j$ :

$$\begin{aligned}\Pi_H^{B_j} &= (V - \frac{1}{2}t)\frac{1}{2} + r\frac{1}{2} + F \\ \Pi_I^{B_j} &= (V - \frac{1}{2}t)\frac{1}{2} - r\frac{1}{2} - F.\end{aligned}\tag{3}$$

The fixed payment  $F$ , which affects only the division of wealth, is determined in bargaining. We assume that the patentee and imitator split the *bargaining surplus*,  $S_B^j = \Pi_T^{B_j} - (\pi_H^j + \pi_I^j)$ , achieved when a contract is implemented in the shadow of regime  $j$ , according to exogenous bargaining power. Firm 1 has bargaining power  $\beta$ , while firm 2 has bargaining power  $1 - \beta$ . Assuming, without loss of generality, that firm 1 is the patentee, efficient bargaining leads to the following payoffs:

$$\begin{aligned}\Pi_H^{B_j} &= \pi_H^j + \beta S_B^j \\ \Pi_I^{B_j} &= \pi_I^j + (1 - \beta)S_B^j.\end{aligned}\tag{4}$$

Hence, each party gets its disagreement payoff plus a share of the extra surplus. The equilibrium value of  $F$  sets the payoffs in (4) equal to those from (3). The same total payoff obtains in equilibrium for each damage regime.

## 2.2. Incentives to Innovate

In our model, the importance of the damage regime emerges in the additional profit that it yields to the patentee. We denote this  $\Delta_{\Pi}^j = \Pi_H^{B_j} - \Pi_I^{B_j}$  and interpret it as the firm's *incentives to innovate*.<sup>26</sup> We assume bargaining power is primitive to the firm—namely, firm

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<sup>25</sup>The firms can implement optimal prices either through price fixing or territorial licensing (quantity fixing). Under price fixing the firms simply agree to  $P_H = P_I = V - \frac{1}{2}t$  and  $r \in \mathcal{R}^*$ . Under territorial licensing, the firms agree to split the consumers into two even territories. As long as  $r \in \mathcal{R}^*$ , neither firm wishes to intrude on the others' territory and prices so that the  $x = \frac{1}{2}$  buyer earns zero utility. This latter conclusion assumes that the firms pick prices conditional on contractually-specified territory, then do not wish to deviate from either the prices or the territory.

<sup>26</sup>For example, in the case of “memoryless” R&D investment technology, an increase in the patentee's payoff that also leads to a wider spread in payoffs yields greater R&D incentives in a patent race setting. For a simple illustration, see section 6 of Anton and Yao (2007). For greater detail, see Reinganum (1989).

1 (which has bargaining power  $\beta$ ) gets share  $\beta$  of the bargaining surplus regardless of whether it is the patentee or imitator.<sup>27</sup> Hence, its incentives to innovate are just the difference in the threat-point payoffs,

$$\Delta_{\Pi}^j = \pi_H^j - \pi_I^j.$$

These incentives depend crucially on the likelihood of enforcement  $\gamma$  and on the damage regime  $j$ . If  $\gamma = 0$ , for example, then  $\Delta_{\Pi}^j = 0$  for all  $j$ . If  $\gamma > 0$ , there are differences across regimes. These are driven primarily by differences in the nature of entry and in *static welfare*, defined as the sum of consumer surplus and joint profit  $\pi_H^j + \pi_I^j$  under threat-point competition. We now consider the reasonable royalty, lost profits and unjust enrichment damage regimes in turn.

### 3. Reasonable Royalty

The reasonable royalty regime awards per-unit and fixed components of damages. To determine the elements of this pair, which we denote  $\{r^*, F^*\}$ , we follow existing US court precedent and use a *hypothetical bargain* over a license,<sup>28</sup> where the components are to be

“...based upon a hypothetical negotiation between the patent owner and the infringer, at the time the infringement began, with both parties to the negotiation assuming that the patent is valid and would be infringed but for the license.”

(*Northlake v. Glaverbel*, 72 F.Supp. 2d 893 [ND Ill. 1999])

Applied to our setting, the precedent demands that  $\{r^*, F^*\}$  be equilibrium outcomes of an arm’s-length bargain. Moreover, the threat-point payoffs  $\pi_H^{RR}$  and  $\pi_I^{RR}$  must reflect the mutual belief that  $\gamma = 1$ .

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<sup>27</sup>That is, if firm 1 were to be the imitator, then the imitator’s payoff would be  $\Pi_I^{Bj} = \pi_I^j + \beta S_B^j$ , in contrast to  $\Pi_I^{Bj}$  in (4). Note that our main results do not change qualitatively if a firm’s bargaining power depends on whether it is the patentee.

<sup>28</sup>Reitzig et al. (2006) argue that the royalty calculation imposed by courts often greatly differs from what the agreed-upon royalty rate would have been. This occurs because courts often impose industry-standard royalty rates and fail to consider other factors (i.e., the cost of designing around the patent, the lack of other licensing opportunities for the patentee, the willingness of the imitator to avoid infringement, etc.) that would affect relative threat points in the bargain. Like Schankerman and Scotchmer (2001), we attempt to remain faithful to the what the law prescribes.

It is easiest to determine recursively the components of the reasonable royalty. We first consider threat points under the *actual bargain*, i.e. the bargain that yields the equilibrium payoffs given in (4). Threat points in the hypothetical bargain obtain for the special case of  $\gamma = 1$ . We then use (3) to solve for  $\{r^*, F^*\}$  and work backwards to compute equilibrium payoffs in the actual bargain.

In the actual bargain, the threat points  $\pi_H^{RR}$  and  $\pi_I^{RR}$  reflect competition in the shadow of both  $\{r^*, F^*\}$  and the uncertain level of patent enforcement  $\gamma$ . If bargaining breaks down, the patentee and imitator compete in prices at arm's length, anticipating that, with probability  $\gamma$ , the court will award the patentee a royalty  $r^*$  on all units sold by the imitator, as well as a fixed payment  $F^*$ . Assuming entry is profitable,<sup>29</sup> this gives rise to the following profit functions, net of expected fixed payments:

$$\begin{aligned}\pi_H^{RR} - \gamma F^* &= \text{Max}_{\{P_H\}} \{R_H(P_H, P_I) + \gamma r^* [D_I(P_H, P_I)]\} \\ \pi_I^{RR} + \gamma F^* &= \text{Max}_{\{P_I\}} \{R_I(P_H, P_I) - \gamma r^* [D_I(P_H, P_I)]\}.\end{aligned}\tag{5}$$

The expected per-unit royalty  $\gamma r^*$  enhances the strategic complementarity of prices, while affecting the pricing decisions of the patentee and imitator in exactly the same way. For the imitator,  $\gamma r^*$  is an artificial marginal cost of additional market share. For the patentee,  $\gamma r^*$  is an artificial opportunity cost of additional market share. In equilibrium, prices are higher than in a standard one-shot Hotelling game but, provided the expected royalty is not too high, market shares are necessarily equal. Hence, total static welfare is maximized. For a sufficiently high expected royalty, there are multiple equilibria and prices may be asymmetric. We have the following result.

**Proposition 1** *Suppose bargaining breaks down and competition ensues in the shadow of the reasonable royalty regime, with components  $\{r^*, F^*\}$ . If  $\gamma r^* \leq V - \frac{3}{2}t$ , the equilibrium is unique, with symmetric prices  $P_H^* = P_I^* = t + \gamma r^*$  and demands  $D_H = D_I = \frac{1}{2}$ . Static welfare is maximized. If  $\gamma r^* \in [V - \frac{3}{2}t, V - t]$ , then the market is fully covered and equilibrium prices*

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<sup>29</sup>Because of the way the hypothetical bargain is implemented, entry is always profitable in equilibrium under reasonable royalty. We show this later in this section.

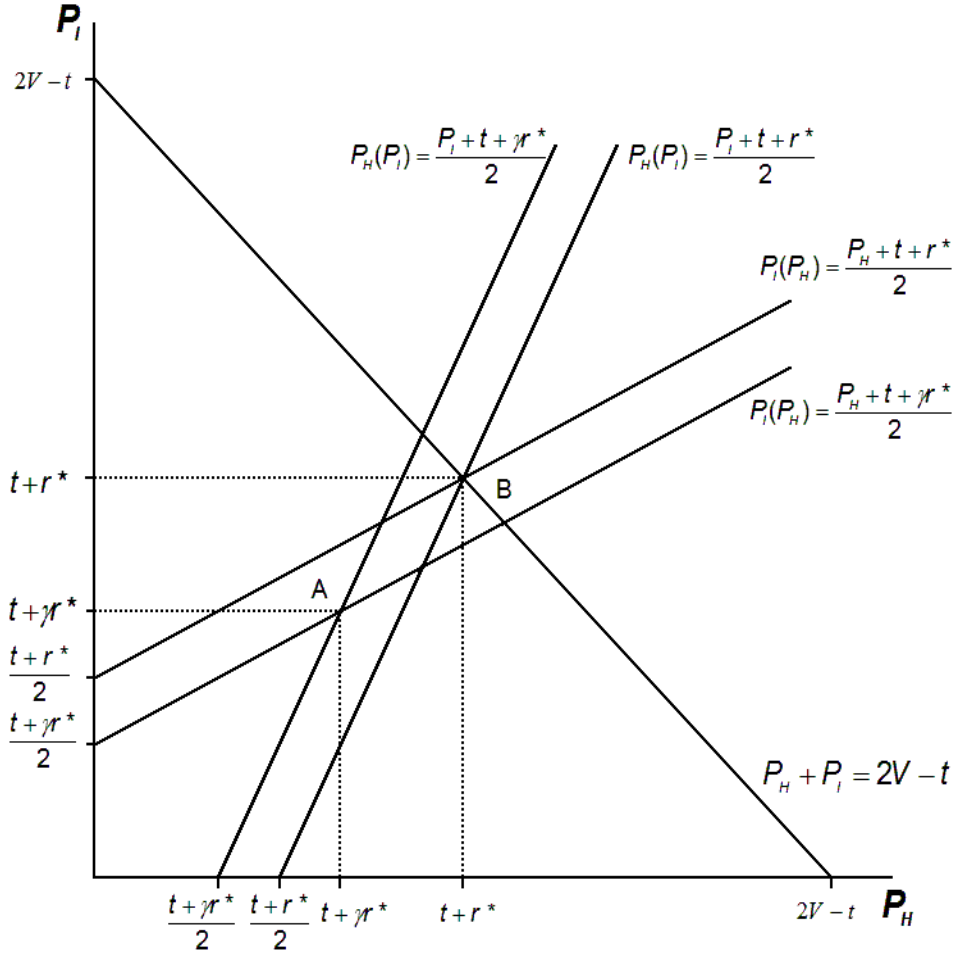


Figure 1: *Threat-Point Competition, Reasonable Royalty,  $r^* = V - \frac{3}{2}t$*

satisfy:

- i.  $P_H + P_I = 2V - t$
- ii.  $P_H \geq \frac{V}{2}$
- iii.  $P_I \geq \frac{V + \gamma r^*}{2}$ .

To see the intuition for the unique-equilibrium case ( $\gamma r^* \leq V - \frac{3}{2}t$ ), consider the symmetric reaction functions:

$$\begin{aligned} P_H(P_I) &= \frac{P_I + t + \gamma r^*}{2} \\ P_I(P_H) &= \frac{P_H + t + \gamma r^*}{2} \end{aligned}$$

These are plotted in Figure 1 for the case  $r^* = V - \frac{3}{2}t$ . For  $\gamma < 1$ , they intersect at prices  $P_H^* = P_I^* = t + \gamma r^*$  (point A). The pivotal buyer, at  $x = \frac{1}{2}$ , receives strictly positive net utility. For  $\gamma = 1$ , they intersect at  $P_H^* = P_I^* = V - \frac{t}{2}$  (point B), and the constraint (1)

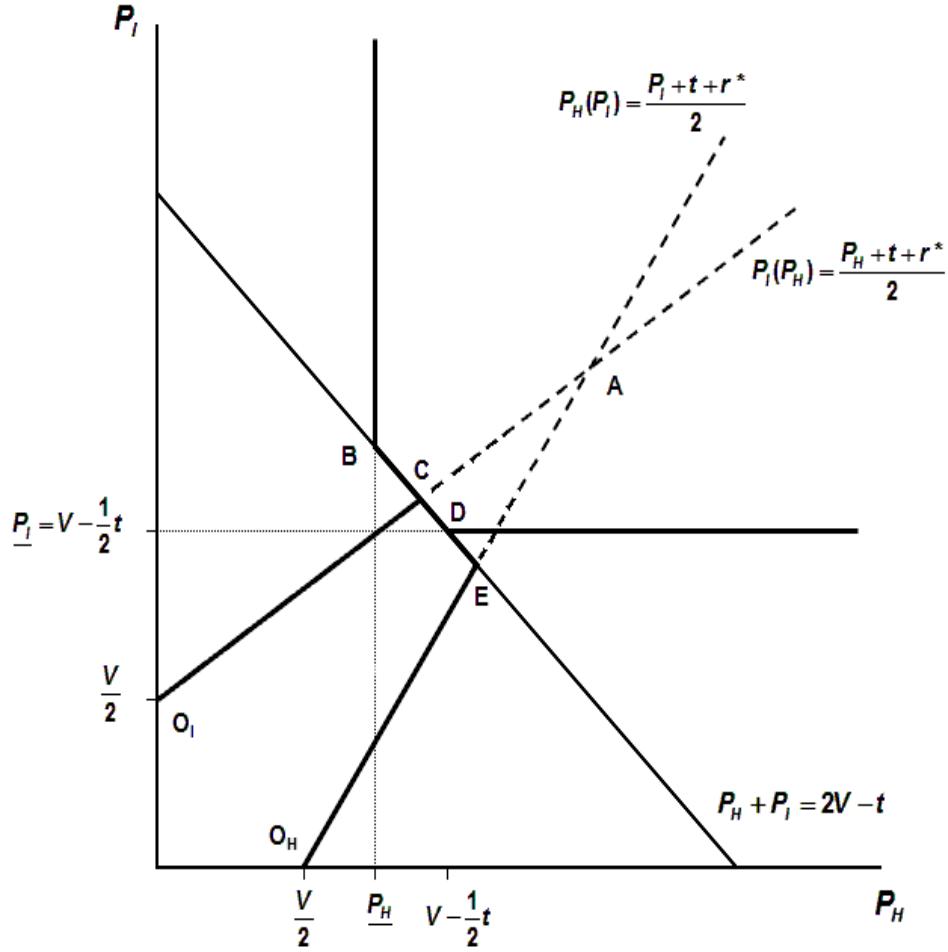


Figure 2: *Threat-Point Competition, Reasonable Royalty,  $r^* = V - t$ ,  $\gamma = 1$*

holds with equality. Hence, if the actual bargain were to break down, prices would be lower than under bargaining if  $\gamma < 1$ . Decreases in the likelihood of patent enforcement mitigate the strategic complementarity in the same way that a lower royalty does, decreasing total profit to  $t + 2\gamma r^*$  and creating a strictly positive bargaining surplus. Since equilibrium market shares remain symmetric, however, transportation costs are minimized and total static welfare is maximized.

If  $\gamma r^* \in (V - \frac{3}{2}t, V - t]$ , then there are multiple equilibria. This case is illustrated in Figure 2, where we set  $r^* = V - t$  and  $\gamma = 1$ . Point A does not constitute an equilibrium, because the prices violate (1). The best response in such situations is either to price on the constraint, in which case the market is covered and the pivotal buyer's net surplus is zero, or, when the other firm's price is uncompetitive (i.e. extremely high), to set price  $\underline{P}_i$ ,

for  $i \in \{H, I\}$ . For the patentee, this is the same as an optimal monopoly price,<sup>30</sup> because marginal changes in  $P_H$  do not affect the size of royalty payments when the imitator's price is uncompetitive. The imitator, however, pays royalties on every unit sold, so  $\underline{P}_I = \frac{V+r^*}{2}$  is optimal when the patentee is uncompetitive.<sup>31</sup>

The constraint forms part of *both* reaction functions. The reaction function for the patentee runs from point  $0_H$  to  $E$ , then from  $E$  to  $B$ , then turns vertical at  $B$ , while for the imitator, it runs from  $0_I$  to  $C$ , then from  $C$  to  $D$ , then turns horizontal at  $C$ . These reaction functions overlap between points  $C$  and  $D$ , and each point in this interval is an equilibrium. As  $r$  increases,  $\underline{P}_I$  also shifts up and, for  $r > V - t$ , the imitator strictly prefers to reduce its market size below  $\frac{1}{2}$ .

### 3.1. The Hypothetical Bargain

By Lemma 1, it is clear that any efficient bargain leads to a per-unit royalty in  $\mathcal{R}^*$ , so the hypothetical bargain must have  $r^* \in \mathcal{R}^*$ . Conditioning on this, we solve for the reasonable royalty by setting the patentee's payoff under the efficient bargain, equation (3), equal to its threat-point payoff plus  $\beta$  times the renegotiation surplus, from equation (4).<sup>32</sup> The hypothetical bargain assumes  $\gamma = 1$  under threat-point competition. Given an  $r^* \in \mathcal{R}^*$ , we have, for the patentee,

$$\left(V - \frac{1}{2}t\right) \frac{1}{2} + \frac{r^*}{2} + F^* = P_H D_H(P_H, P_I) + r^* D_I(P_H, P_I) + F^* + \beta \left[ \frac{1}{2t} (P_H - P_I)^2 \right], \quad (6)$$

where  $P_H, P_I, D_H$  and  $D_I$  reflect an equilibrium under (threat-point) competition with per-unit royalty  $r^*$ , and the bargaining surplus is obtained using (2). It is immediately clear from (6) that  $F^*$  is not unique, a point of similarity between our setting and that of Schankerman and Scotchmer (2001) that we return to later.

Canceling  $F^*$  from both sides of the equals sign, we turn our attention to identifying  $r^*$ . From Proposition 1, it is clear that there is only one  $r \in \mathcal{R}^*$  for which threat-point

<sup>30</sup>Monopoly pricing is discussed in detail in section 4.

<sup>31</sup>Intuitively, the royalty functions like a marginal cost for the imitator and, for  $r^*$  (or any  $r \in \mathcal{R}^*$ ), if the patentee's price is uncompetitive, it is optimal for the imitator to serve less than the entire market.

<sup>32</sup>Using the imitator's payoffs yields the same thing.

equilibrium prices and demands are unique and which satisfies (6),  $r = r^U = V - \frac{3}{2}t$  (recall Figure 1). Clearly,  $r^* = r^U$  is reasonable.

For the other  $r \in \mathcal{R}^*$ , the prices that the parties expect to obtain in equilibrium under threat-point competition matter for the royalty that they will agree to. If they anticipate the symmetric equilibrium,  $P_H = P_I = V - \frac{1}{2}t$ , then there is no bargaining surplus and (6) is trivially satisfied for all  $r \in \mathcal{R}^*$ . For asymmetric prices, however, the bargaining surplus is positive. In this case, (6) is satisfied for some, but not all,  $r \in \mathcal{R}^*$ , and the royalties that satisfy the condition depend on  $\beta$ .<sup>33</sup> Since threat-point payoffs are assumed to result from arm's-length competition, where price-fixing does not occur, it is awkward to consider  $r$  “reasonable” when it satisfies the requirements of the hypothetical bargain for some, but not all, pricing equilibria. It is also awkward for the threat-point pricing equilibrium implied by a particular  $r^*$  to depend on  $\beta$ , as bargaining power is actually irrelevant under threat-point competition.<sup>34</sup> The following result offers an appealing refinement.

**Proposition 2.** *The only per-unit royalty that satisfies the requirements of the hypothetical bargain for all implied threat-point equilibria and  $\beta$  is  $r^U = V - \frac{3}{2}t$ .*

In analyzing the reasonable royalty regime, we henceforth restrict attention to  $r^U$  (where the superscript denotes “unique”).

It remains to identify  $F^*$ . Consider again the hypothetical bargain. If the patentee and imitator were to believe that  $\gamma = 1$  if bargaining were to break down, then the threat-point payoffs would be, for  $r^* = r^U$ ,

$$\begin{aligned}\pi_H^{RR^*} &= V - t + F^* \\ \pi_I^{RR^*} &= \frac{t}{2} - F^*,\end{aligned}\tag{7}$$

provided that both payoffs are non-negative. If one of these payoffs were negative, then the

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<sup>33</sup>Writing demands as a function of  $P_H$  and  $P_I$  and using the restriction  $P_H + P_I = 2V - t$ , it can be shown that condition (6) is equivalent to  $r^* = V - t + (\beta - \frac{1}{2})(P_I - P_H)$  and  $P_H$  and  $P_I$  are equilibrium prices given per-unit royalty  $r^*$ . For a given threat-point equilibrium, this condition rules out many per-unit royalties in  $\mathcal{R}^*$ . For example, if the parties in the hypothetical bargain anticipate asymmetric prices under threat-point competition and  $\beta \neq \frac{1}{2}$ , then  $r^* = V - t$  is not reasonable. On the other hand, if  $\beta = \frac{1}{2}$ , then  $r^* = V - t$  is reasonable for any asymmetric prices, but no other royalty is reasonable.

<sup>34</sup>In essence, the hypothetical bargain does not make much sense if there is bargaining surplus, because the fixed component  $F^*$  is part of the threat-point payoff and therefore cannot serve as a means of sharing the bargaining surplus.

firm facing such a payoff would choose to stay out of the market and earn a payoff of 0. However, any  $F^*$  that results in a negative payoff in (7) cannot satisfy the requirements of the hypothetical bargain.

**Proposition 3** *The reasonable fixed component  $F^* \in [-(V - t), \frac{t}{2}] \equiv \mathcal{F}^*$*

Because the bargaining surplus is zero, the payoffs in (7) are identical to the equilibrium payoffs under the efficient bargain with per-unit royalty  $r^U$  and fixed payment  $F^*$ , given by equation (3). Clearly, the patentee and imitator would never agree to an  $F^*$  such that either of those payoffs is negative. Conditional upon this restriction, however, any  $F^*$  can be an equilibrium in the hypothetical bargain, precisely because the payoffs above are identical to those in (3). The logic of this result is essentially the same as in Schankerman and Scotchmer (2001). The court's choice of  $F^*$  determines the fixed component of the license fee, so there is a circularity—multiple values of  $F^*$  satisfy the requirement.

### 3.2. Equilibrium Payoffs and Incentives to Innovate

Now consider the actual bargain with reasonable royalty  $\{r^U, F^*\}$ . In equilibrium, the fixed payment  $F$  equates (4) and (3), yielding

$$F = \gamma F^* + (1 - \gamma)(\beta - 1) \left( V - \frac{3}{2}t \right),$$

and incentives to innovate

$$\Delta_{\Pi}^{RR} = \gamma \left( V - \frac{3}{2}t + 2F^* \right).$$

We have the following result.

**Proposition 4** *Suppose  $r^* = r^U$  and  $\gamma < 1$ . If  $F^* > 0$ , then the equilibrium fixed payment  $F < F^*$ . This payment equals  $F^*$  if and only if  $F^* = F^R \equiv (\beta - 1) \left( V - \frac{3}{2}t \right) \leq 0$ . Incentives to innovate are maximized for  $F^* = F^{Max} \equiv \frac{t}{2}$ .*

Clearly, if  $\gamma = 1$ , then  $F = F^*$ , as the hypothetical bargain requires. Otherwise, the parties

reduce the size of the fixed payment in accordance with the weaker bargaining position of the patentee and stronger bargaining position of the imitator.

The only reasonable fixed component for which the actual bargain produces  $F^*$  as the equilibrium fixed payment is  $F^* = F^R$  (where the superscript “R” denotes “rational”). While rationality is an appealing property for  $F^*$ , employing  $F^R$  yields peculiar results. First, it implies a negative fixed payment. Second, it yields incentives  $\Delta^{RR} = (2\beta - 1) \left( V - \frac{3}{2}t \right)$ , which are positive if and only if  $\beta > \frac{1}{2}$ .

It is clear that incentives to innovate are maximized for the largest value that satisfies the requirements for the hypothetical bargain,  $F^* = F^{Max}$ . Hence, the reasonable royalty regime with components  $\{r^U, F^{Max}\}$  yields incentives

$$\Delta_{\Pi}^{RR} = \gamma \left( V - \frac{1}{2}t \right).$$

As  $V$  increases,  $\Delta_{\Pi}^{RR}$  increases at rate  $\gamma$ .

#### 4. Lost Profits

The lost profits regime assumes that the rightful profit for the patentee is a monopoly profit, and sets damages equal to the difference between monopoly profit and duopoly profit. Consider first the monopoly case. A consumer located at point  $x$  on the line will buy from the patentee-monopolist if  $P_M \leq V - tx$  (the subscript  $M$  denotes “monopolist”). The optimal price depends on the value of the product, relative to transportation costs:

$$P_M = \begin{cases} V - t & \text{if } V > 2t \\ \frac{V}{2} & \text{if } V \leq 2t. \end{cases}$$

For sufficiently high  $V$ , the monopolist serves the entire market, and holds the consumer at  $x = 1$  to his reservation utility, net of transportation costs. In this case,  $\pi_M = V - t$ . Otherwise, the monopolist serves only consumers at  $x \leq \frac{V}{2t}$ , and prices so that marginal revenue for the pivotal buyer equals zero. In this case,  $\pi_M = \frac{V^2}{4t}$ .

Expected profit functions under competition in the shadow of lost profits are

$$\begin{aligned}\pi_H^{LP} &= \text{Max}_{\{P_H\}} \{R_H(P_H, P_I) + \gamma [\pi_M - R_H(P_H, P_I)] 1(\pi_M > R_H(P_H, P_I))\} \\ \pi_I^{LP} &= \text{Max}_{\{P_I\}} \{R_I(P_H, P_I) - \gamma [\pi_M - R_H(P_H, P_I)] 1(\pi_M > R_H(P_H, P_I))\},\end{aligned}$$

where  $1(\cdot)$  is the indicator function. Passive infringement occurs whenever the imitator makes positive sales but these indicator functions are turned off. This is possible for  $V \in [\frac{3}{2}t, 2t)$ , as the patentee-monopolist does not cover the entire market. The imitator can charge up to  $P_I^* = \frac{3V}{2} - t$  (covering the remainder of the market), and not introduce lost profits. For  $\gamma < 3 - \frac{9t}{2V}$ , however, this does not hold as an equilibrium.

**Proposition 5** *Let  $V \geq \frac{9t}{2(3-\gamma)}$ . Under the lost profits regime, if the imitator competes, the equilibrium is unique. Prices and demands are not symmetric. The patentee charges a lower price and serves a higher demand than the imitator. Passive infringement, with positive sales by the imitator, is not an equilibrium.*

Unlike the reasonable royalty case, the reaction functions are not symmetric,

$$\begin{aligned}P_H(P_I) &= \frac{P_I + t}{2} \\ P_I(P_H) &= \frac{(1+\gamma)P_H + t}{2},\end{aligned}$$

so the equilibrium prices and demands are also asymmetric:

$$\begin{aligned}P_H^* &= \frac{3t}{3-\gamma} & D_H(P_H^*, P_I^*) &= \frac{3}{6-2\gamma} \\ P_I^* &= \frac{(3+\gamma)t}{3-\gamma} & D_I(P_H^*, P_I^*) &= \frac{3-2\gamma}{6-2\gamma}.\end{aligned}\tag{8}$$

Passive infringement does not obtain because the imitator's optimal reaction to  $P_H = P_M$  is to price more aggressively than under passive infringement.<sup>35</sup> The equilibrium in (8) is unique for all  $\gamma$  when  $V \geq \frac{9}{4}t$ , so for sufficiently high values of  $V$ , the monopolist covers the entire market and there is a unique, full-coverage equilibrium under duopoly. The patentee charges a lower price than the imitator, and captures more than half of the market. Both prices are increasing in  $\gamma$ , and may exceed  $P_M$ .

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<sup>35</sup>If the  $V \geq \frac{9t}{2(3-\gamma)}$  condition is violated, passive infringement may be an equilibrium, but is not typically unique. We consider this case later in the section.

These results are driven by the strategic complementarity of prices. Just as with reasonable royalty damages, the likelihood of patent enforcement,  $\gamma$ , enhances this complementarity under the lost profits regime, driving prices higher. Effectively, the imitator's only way to minimize the patentee's lost profits is to reduce its market share by raising its price. In response, the patentee raises its price. For sufficiently high  $\gamma$ , the complementarity effect dominates the price-suppressing effects of competition, and prices may exceed  $V - t$ .<sup>36</sup>

The profits under the equilibrium in (8) are:

$$\begin{aligned}\pi_H^{LP*} &= \frac{9t(1-\gamma)}{2(3-\gamma)^2} + \gamma(\pi_M) \\ \pi_I^{LP*} &= \frac{9t+6\gamma t-2t\gamma^2}{2(3-\gamma)^2} - \gamma(\pi_M)\end{aligned}\tag{9}$$

While the patentee's profit cannot be higher under duopoly than under monopoly, total profits may be higher than under monopoly. The complementarity-enhancing effect of  $\gamma$  is to promote, essentially, more effective price discrimination by the firms.

Increases in  $\gamma$  also increase expected damages, but the imitator's expected profit may nonetheless increase with  $\gamma$  because price-complementarity effects may dominate for high  $\gamma$ . The difference in profits is also not a monotone function of  $\gamma$ , and it is possible that the difference in profits is maximized for an interior value of  $\gamma$ . Hence, incentives to innovate may be highest for less-than-perfect patent enforcement.

Each point is illustrated in Figure 3, for the case  $V = \frac{9}{4}$ ,  $t = 1$ . Total profit under duopoly is higher than monopoly profit ( $\frac{5}{4}$ ) for all  $\gamma > .46$  (approximately). The imitator's profit is minimized at around  $\gamma = .66$ . The difference in profits is maximized at around  $\gamma = .88$ .

When  $V$  is very high, relative to  $t$ , it may be unprofitable for an imitator to enter.

**Corollary 1** *Under the lost profits regime, for any positive  $\gamma$ , the imitator stays out of the market for sufficiently high  $V > V^{NE}(\gamma) > 2t$ , in which case the patentee earns  $\pi_M = V - t$ .*

As  $V$  increases, lost profit damages increase but revenues do not, so the imitator's expected profit under entry falls and becomes negative once  $V$  exceeds  $V^{NE}$ , where the latter is defined by  $\pi_I^{LP} = 0$  from (9).

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<sup>36</sup>The patentee's price, which is the lower of the two prices, is higher than  $P_M$  whenever  $\gamma > \frac{3V-6t}{V-t}$ .

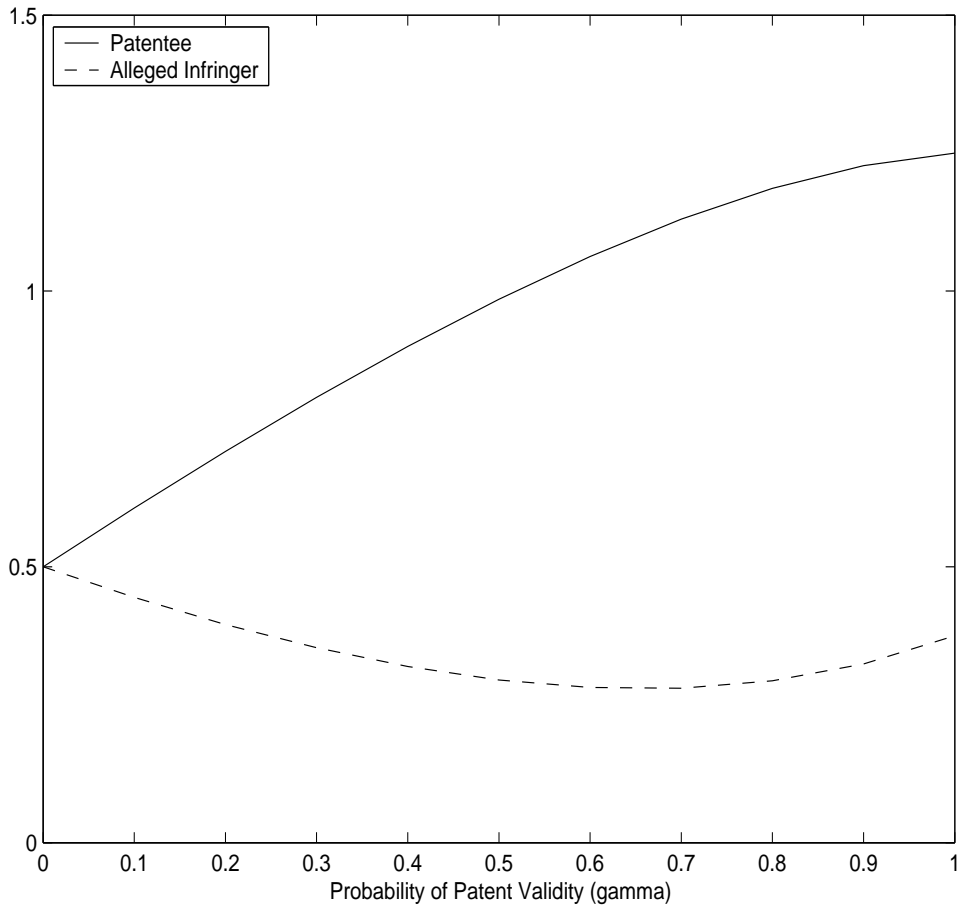


Figure 3: *Duopoly Profits, Lost Profits Regime,  $V = \frac{9}{4}, t = 1$*

For products of sufficiently low value ( $V < 2t$ ), passive infringement is possible. As in the model of Anton and Yao (2007), infringement is never deterred in such cases. When the assumption in Proposition 5 does not hold, the cutoff buyer is not willing to buy from either seller at the prices in (8), so each seller has the incentive to cut price to cover the market. In this case, there are multiple equilibria, and passive infringement may be among the set. Thus, it appears that passive infringement as a unique equilibrium under the lost profits regime is a phenomenon of process patents.

Consider a low- $V$  example, with  $V = \frac{3}{2}$  and  $t = 1$ . Under monopoly, the patentee sets  $P_M = \frac{3}{4}$ , and captures  $\frac{3}{4}$  of the market, earning a profit of  $\frac{9}{16}$ . In the standard Hotelling duopoly model with these values, there is a unique equilibrium in prices,  $P_H^* = P_I^* = 1$ , where (1) holds with equality. This corresponds to the case  $\gamma = 0$  in our duopoly setting.

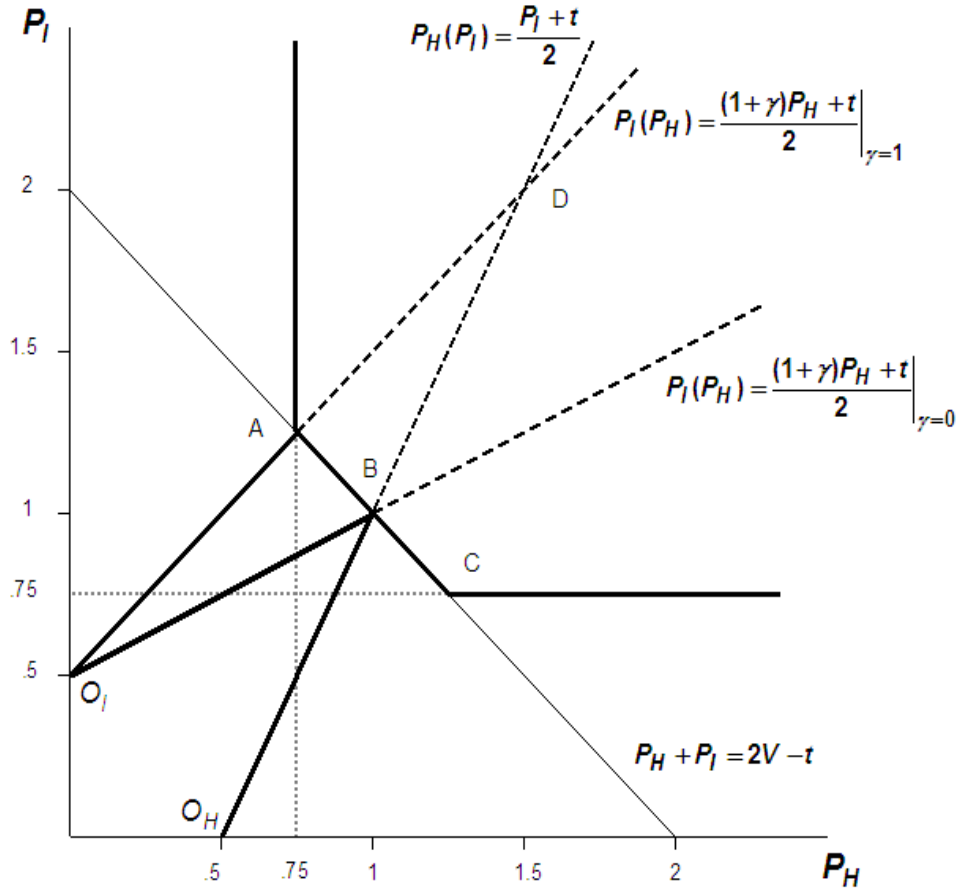


Figure 4: *Reaction Functions, Lost Profits Regime,  $V = \frac{3}{2}, t = 1$*

In this equilibrium, the patentee and the imitator split the market evenly and earn identical profits of  $\frac{1}{2}$ , so lost profits are  $\frac{1}{16}$ . Because  $\gamma = 0$ , the patentee never recovers lost profits.

This is easily seen in Figure 4, which plots the firms' reaction functions and constraint (1). Point B represents the equilibrium for the  $\gamma = 0$  case, while point A represents prices consistent with passive infringement. It is obvious that passive infringement is not an equilibrium. If the patentee were to price as a monopolist, the imitator gains by lowering price below  $\frac{5}{4}$ , to the level of its reaction function, and capturing more market share.

Just as with Figure 2 earlier, the standard reaction functions  $P_H(P_I)$  and  $P_I(P_H)$  are correct only if condition (1) is satisfied. Once they reach the constraint, each firm's best response is to price so that the market is just covered, so long as its price does not fall below the monopoly price  $\frac{3}{4}$ . Thus, the reaction functions have three linear pieces. In the figure,

the patentee's reaction function runs from point  $0_H$  to  $B$ , then from  $B$  to  $A$ , then from  $A$  turns vertical, while the imitator's reaction function runs from  $0_I$  to  $B$ , then from  $B$  to  $C$ , then from  $C$  turns horizontal.

As  $\gamma$  rises, the imitator has a stronger incentive to minimize lost profits by raising its price. As a result, its reaction function is steeper. Since the new reaction function crosses the patentee's reaction function at prices (found in (8)) that violate (1), point  $D$  is not an equilibrium. For  $\gamma = 1$ , the imitator's reaction function runs from point  $0_I$  to  $A$ , then from  $A$  to  $C$ , then from  $C$  turns horizontal. In this case, there is a continuum of price combinations, between points  $A$  and  $B$  on the constraint, that form equilibria.<sup>37</sup> Passive infringement is the only one of these equilibria in which there are no lost profits.

#### 4.1. Equilibrium Payoffs and Incentives to Innovate

Restricting attention to  $V \geq \frac{9t}{2(3-\gamma)}$ , equilibrium payoffs depend on whether the imitator enters and competes. In the former case, the threat points are given by (9), and we have:<sup>38</sup>

$$\Delta_{\Pi}^{LP}(\text{Entry}) = \frac{2\gamma^2 t - 15\gamma t}{2(3-\gamma)^2} + 2\gamma(\pi_M).$$

If instead  $V > V^{NE}$ , then the imitator does not enter, so the patentee earns  $\pi_M$  and the imitator earns 0. Since  $V^{NE} > 2t$ , we have that  $\pi_M = V - t$ . Hence,

$$\Delta_{\Pi}^{LP}(\text{No Entry}) = V - t.$$

It is easy to show that at  $V = V^{NE}$ ,  $\Delta_{\Pi}^{LP}(\text{No Entry}) > \Delta_{\Pi}^{LP}(\text{Entry})$ , so the patentee's profit experiences a discrete positive jump at the point where entry is precluded.<sup>39</sup> As  $V$  increases further,  $\Delta_{\Pi}^{LP}$  increases at rate 1, faster than  $\Delta_{\Pi}^{RR}$  increases with  $V$ .

<sup>37</sup>Note, for instance, that  $P_H^* = P_I^* = 1$  remains an equilibrium.

<sup>38</sup>One can use (4) and (3) to determine  $F$ . It is of no specific interest here, so we omit the tedious calculations.

<sup>39</sup> $V = V^{NE}$  implies that  $\pi_I^{LP} = 0$ , so  $\Delta_{\Pi}^{LP} = \frac{9t(1-\gamma)}{2(3-\gamma)^2} + \gamma(V - t)$ . This is smaller than  $V - t$  if  $V > \frac{27t-12\gamma t-2\gamma^2 t}{2(3-\gamma)^2}$ . Since the right-hand side is smaller than  $V^{NE}$  (see the proof of Corollary 1 in the appendix), the condition holds.

## 5. Unjust Enrichment

Under unjust enrichment, the patentee receives the entire revenue earned by the imitator if its patent is found valid and infringed. Expected profit functions are:

$$\begin{aligned}\pi_H^{UR} &= \text{Max}_{\{P_H\}} \{R_H(P_H, P_I) + \gamma R_I(P_H, P_I)\} \\ \pi_A^{UR} &= \text{Max}_{\{P_I\}} \{R_I(P_H, P_I) - \gamma R_I(P_H, P_I)\}.\end{aligned}$$

Pricing incentives for the patentee and imitator, under this regime, are the mirror images of those in the lost profits case.

**Proposition 6** *Let  $V \geq \frac{9t}{2(3-\gamma)}$ . Under the unjust enrichment regime, the equilibrium is unique. Prices and demands for the patentee and imitator are reversed from the lost profits equilibrium — the imitator charges a lower price and serves a higher demand than the patentee. Profits are always nonnegative and are never a function of  $V$ .*

This follows immediately from the fact that the profit functions, apart from a level shift due to expected lost profit damages  $\gamma\pi_M$ , are exactly reversed from the lost profits case.<sup>40</sup> Equilibrium prices and demands are:

$$\begin{aligned}P_H^* &= \frac{(3+\gamma)t}{3-\gamma} & D_H(P_H^*, P_I^*) &= \frac{3-2\gamma}{6-2\gamma} \\ P_I^* &= \frac{3t}{3-\gamma} & D_I(P_H^*, P_I^*) &= \frac{3}{6-2\gamma}.\end{aligned}$$

This set of findings is quite similar to those in Choi (2006) and Anton and Yao (2007), who find an analogous reversal under quantity competition. Now, the patentee has the greater incentive to raise price because its expected profit increases as the gross profit of the imitator increases. The imitator's expected payoff is proportional to the case where there is no patent (or zero enforcement), so it prices normally. In equilibrium, the imitator garners more than half of the market. Profits are independent of  $V$ :

$$\begin{aligned}\pi_H^{UR*} &= \frac{9t+6\gamma t-2t\gamma^2}{2(3-\gamma)^2} \\ \pi_I^{UR*} &= \frac{9t(1-\gamma)}{2(3-\gamma)^2}.\end{aligned}\tag{10}$$

The imitator always earns a non-negative profit, so infringement is never deterred.

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<sup>40</sup>Similarly, there are multiple equilibria for  $V \in [\frac{3t}{2}, \frac{9t}{2(3-\gamma)}]$ .

|   | <b>Reasonable<br/>Royalty</b><br>$\{r^U, F^{Max}\}$ | <b>Lost<br/>Profits</b><br>$(V \leq V^{NE})$                      | <b>Lost<br/>Profits</b><br>$(V > V^{NE})$ | <b>Unjust<br/>Enrichment</b>                     |
|---|---|---|---|--|
| <b>Threat-Point<br/>Demands<br/>Symmetric?</b>                | Yes   | No  | No  | No   |
| <b>Threat-Point<br/>Entry?</b>                                | Yes   | Yes   | No  | Yes  |
| <b>Incentives to<br/>Innovate <math>\Delta_{\Pi}^j</math></b> | $\gamma \left( V - \frac{1}{2}t \right)$            | $\frac{2\gamma^2 t - 15\gamma t}{2(3-\gamma)^2} + 2\gamma(\pi_M)$ | $V - t$                                   | $\frac{15\gamma t - 2\gamma^2 t}{2(3-\gamma)^2}$ |

Table 1: *Summary of Results*,  $V \geq \frac{9t}{2(3-\gamma)}$

### 5.1. Equilibrium Payoffs and Incentives to Innovate

Using the threat points for competition in the shadow of unjust enrichment defined in (10), we find:

$$\Delta_{\Pi}^{UR} = \frac{15\gamma t - 2\gamma^2 t}{2(3-\gamma)^2}$$

It is easily seen that  $\frac{d\Delta_{\Pi}^{UR}}{dV} = 0$ —under the unjust enrichment regime, the reservation value  $V$  has no effect on the incentives to innovate. This contrasts with the positive effect of  $V$  on  $\Delta_{\Pi}^{RR}$  (rate  $\gamma$ ) and  $\Delta_{\Pi}^{LP}$  (rate 1). Hence, for valuable products,  $\Delta_{\Pi}^{UR}$  is small relative to the incentives under the other damage regimes.

## 6. Comparison of Regimes

In equilibrium, there is no difference in static welfare under the various regimes, as the prices set under efficient bargaining are the same. The key differences pertain to characteristics of threat-point competition. Most importantly, because a firm with bargaining power  $\beta$  earns the same share of the bargaining surplus in equilibrium regardless of whether it is the patentee or imitator, the difference in threat-point payoffs measures exactly the incentives to innovate and patent. Additionally, these payoffs reflect what would happen if bargaining

were to break down or never take place (if, say, transactions costs are high), something that often occurs in practice.

Results for the unique equilibrium case of  $V \geq \frac{9t}{2(3-\gamma)}$  are summarized in Table 1. The reasonable royalty regime is the only one that yields symmetric demands under threat-point competition, while the lost profits regime is the only one that may deter infringement. Each of these characteristics improves incentives to innovate.

With symmetric demands, the maximum level of surplus is generated under threat-point competition. When  $\gamma$  is high under the reasonable royalty regime, equilibrium threat-point prices are near the collusive level, so the firms' profits are nearly equal to the maximum joint profit. With the fixed fee set at  $F^{Max}$ , nearly all of the imitator's profit is transferred to the patentee. Hence, the difference in the threat-point payoffs, which equal incentives to innovate, are high. As  $\gamma$  approaches 1, the patentee receives the entire collusive profit, so incentives are *maximized*. Generally, then, the reasonable royalty regime with  $\{r^U, F^{Max}\}$  generates high incentives when  $\gamma$  is high, and low incentives when  $\gamma$  is low.

On the other hand, the lost profits regime is the only regime that may deter infringement. For  $V > V^{NE}$ , the imitator's expected payoff from competing in the shadow of the lost profits regime is negative, so it chooses to stay out of the market. When this happens, the patentee earns the monopoly payoff,  $V - t$ , while the imitator earns nothing. Incentives to innovate equal the patentee's payoff. For  $\gamma = 1$ , this is always smaller than the incentives under reasonable royalty  $\{r^U, F^{Max}\}$ . For any  $\gamma < 1$ , however, the lost profits regime generates the highest incentives to innovate for sufficiently high  $V$ .

Even if the lost profits regime does not preclude infringement (e.g., if  $\gamma$  is low), the incentives to innovate still grow most quickly with  $V$  under this regime. For  $V \geq 2t$ , we have  $\pi_M = V - t$ , so  $\Delta_{\Pi}^{LP}$  increases with  $V$  at rate  $2\gamma$ . In contrast,  $\Delta_{\Pi}^{RR}$  increases with  $V$  at rate  $\gamma$ , and  $\Delta_{\Pi}^{UR}$  does not change with  $\gamma$ . Hence,  $\Delta_{\Pi}^{LP}$  is highest for sufficiently large  $V$ .<sup>41</sup>

Unjust enrichment does not generate symmetric demands or deter entry, so it is a poor mechanism for generating incentives to innovate. Since  $\Delta_{\Pi}^{UR}$  does not increase with  $V$ , it generates particularly poor incentives for innovating valuable products. The following

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<sup>41</sup>If  $V < 2t$ , then  $\Delta_{\Pi}^{LP}$  grows more slowly with  $V$ , but still faster than  $\Delta_{\Pi}^{RR}$  and  $\Delta_{\Pi}^{UR}$ . Recalling that  $\pi_M = \frac{V^2}{4t}$  in that case, we have  $\frac{d\Delta_{\Pi}^{LP}}{dV} = 2\gamma \left(\frac{V}{2t}\right) = \gamma \left(\frac{V}{t}\right)$ , which exceeds  $\gamma$  when  $V > \frac{3t}{2}$ .

summarizes our results.

**Proposition 7.** *Let  $V \geq \frac{9t}{2(3-\gamma)}$  and suppose the reasonable royalty regime uses components  $\{r^U, F^{Max}\}$ . For  $\gamma \in (0, 1)$ , we have the following. The unjust enrichment regime fails to generate the highest incentives to innovate. If the lost profits regime does not preclude entry, then it generates the highest incentives to innovate for*

$$V > \frac{3t}{2} + \frac{15t - 2\gamma t}{2(3 - \gamma)^2}. \quad (11)$$

*If the lost profits regime does preclude entry, then it generates the highest incentives to innovate for*

$$V > \frac{(2 - \gamma)t}{2(1 - \gamma)}. \quad (12)$$

*If either case holds, we have  $\Delta_{\Pi}^{LP} > \Delta_{\Pi}^{RR} > \Delta_{\Pi}^{UR}$ . If  $\gamma = 1$ , the the reasonable royalty regime generates the highest incentives to innovate for any  $V$ .*

Figure 5 shows which regime provides the highest incentives to innovate for all  $\gamma$  and  $\frac{V}{t}$ . The gray area covers combinations where  $V < \frac{9t}{2(3-\gamma)}$ , i.e. where equilibria are not unique under the lost profits or unjust enrichment regimes. Just above this region the reasonable royalty regime is best, while higher still the lost profits regime is best. The curve separating the top two regions is not smooth. For low  $\gamma < \hat{\gamma}$ , the lost profits regime may generate higher incentives to innovate even if it does not preclude infringement, and the cutoff follows (11). For higher  $\gamma$ , the reasonable royalty regime yields incentives high enough so that the lost profits regime dominates only if it precludes infringement, and the cutoff follows (12). The middle (jag-tooth) section of the curve, for  $\gamma \in [\hat{\gamma}, \hat{\hat{\gamma}}]$  is precisely the cutoff  $\frac{V^{NE}}{t}$ .<sup>42</sup>

If  $F^* < F^{Max}$ , the incentives under the reasonable royalty regime decrease, shifting the top curve down. The rankings  $\Delta_{\Pi}^{LP} > \Delta_{\Pi}^{RR} > \Delta_{\Pi}^{UR}$  from Proposition 7 continue to hold for

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<sup>42</sup>Note that, in cases where the reasonable royalty regime generates the highest incentives to innovate, the unjust enrichment regime may generate higher incentives than the lost profits regime. The clearest case is when the lost profits regime fails to deter entry, but  $\gamma$  is high. Because of the location economies in this model, there are efficiency gains from duopoly relative to monopoly. Under the lost profits regime, the imitator gets to keep some of these gains, even for  $\gamma = 1$ . Under unjust enrichment, however, the imitator expects to surrender virtually all of them when  $\gamma$  is high. Since total profit is identical under the lost profits (with entry) and unjust enrichment regimes, it follows that the patentee is better off under the latter for high  $\gamma$ .

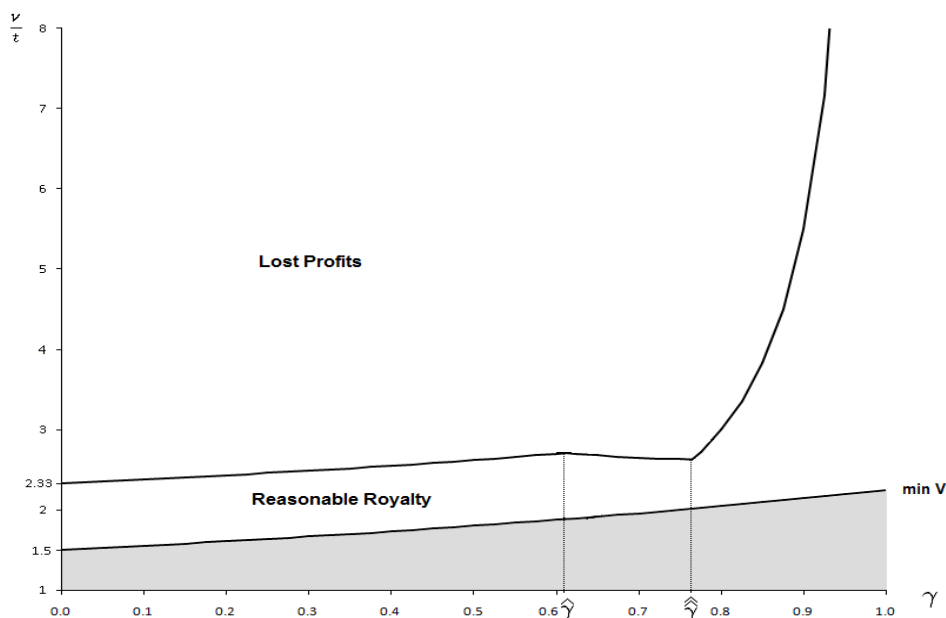


Figure 5: *Highest Incentives to Innovate*,  $V \geq \frac{9t}{2(3-\gamma)}$

(lower) sufficiently high values of  $V$ . While starting at a lower level, incentives under the reasonable royalty regime still increase with  $V$  at rate  $\gamma$ .

Clearly, our results yield a strong efficiency argument against the use of the unjust enrichment regime. It fails to provide strong R&D incentives and fails to yield market efficiency if bargaining breaks down. There is a third, subtle reason why it is inappropriate. In the equilibrium, the patentee has a stronger incentive to raise his price because doing so increases the damages he expects to receive. In general (though no patent cases could be found that discussed this issue), courts require plaintiffs in civil suits to mitigate damages.<sup>43</sup> The behavior under the unjust enrichment regime is perverse, in light of this.

## 7. Conclusion

Focusing on product patents in a differentiated, duopoly setting, we find that the reasonable royalty regime gives the court significant discretion in assigning damages. If it chooses components to maximize incentives to innovate and patent enforcement is certain, then the

<sup>43</sup>For instance, if a buyer breaches a contract to purchase tomatoes, the seller must try to sell them to others before they rot.

reasonable royalty regime generates higher incentives than the other two regimes. If patent enforcement is uncertain, then the lost profits regime results in the biggest difference in profits for extremely valuable patents, as it is the only regime that may deter infringement. Contrary to recent work on process patents, we do not typically find unique equilibria characterized by passive infringement. The unjust enrichment regime provides poor incentives to innovate for valuable products and does not yield market efficiency, making it the weakest of the three.

The reasonable royalty regime seems to be the easiest regime for US courts to implement, but lost profit damages are not uncommon. In a study of published damage awards over 1982-92, Coolley (1993) finds that a reasonable royalty rate was set in 65 cases, lost profits were awarded in 40 cases, and a combination of the two remedies was used in 19 cases. The application of lost profit damages yielded higher payoffs, on average, consistent with the general perception of patentees.

In practice, setting a damage award for lost profits is as much of an art as it is a science, but it clearly relies on economic benchmarks. Focusing on quantity competition, Werden, Froeb and Beavers (1999) argue that many courts award damages incorrectly, either by ignoring “price erosion,” or by allowing for price erosion but ignoring “quantity accretion.” When products are differentiated, prices do not necessarily fall with competition. Indeed, we find that prices may be higher than the monopoly price under the lost profits or unjust enrichment regimes. Thus, our results may also help to inform the use of simulation models (see Werden, Froeb and Langenfeld 2000), or related empirical examination of the current court system. We look forward to further progress.

## Appendix

**Proof of Lemma 1.** It is clear from (2) that total profit is maximized only if  $P_H = P_I = V - \frac{1}{2}t$ . We now identify royalties such that this is a self-reinforcing equilibrium.

Consider first the case where (1) holds strictly. The profit functions are

$$\begin{aligned}\pi_H^{RR} &= \frac{P_H}{2} + \frac{P_H P_I - P_H^2}{2t} + r \left( \frac{1}{2} + \frac{P_H - P_I}{2t} \right) + F \\ \pi_I^{RR} &= \frac{P_I}{2} + \frac{P_H P_I - P_I^2}{2t} - r \left( \frac{1}{2} + \frac{P_H - P_I}{2t} \right) - F,\end{aligned}\tag{13}$$

and yield the following first-order conditions:

$$\begin{aligned}P_H(P_I) &= \frac{P_I + t + r}{2} \\ P_I(P_H) &= \frac{P_H + t + r}{2}.\end{aligned}\tag{14}$$

Solving these reaction functions yields the equilibrium:

$$\begin{aligned}P_H^* &= t + r & D_H(P_H, P_I) &= \frac{1}{2} & \pi_H^* &= \frac{1}{2}t + r \\ P_I^* &= t + r & D_I(P_H, P_I) &= \frac{1}{2} & \pi_I^* &= \frac{1}{2}t.\end{aligned}\tag{15}$$

Conditional on the prices above, (1) is a strict inequality if and only if  $r < V - \frac{3}{2}t$ . In such cases, prices are strictly lower than the efficient level.

As  $r$  approaches  $V - \frac{3}{2}t$  from below, (1) approaches equality, and prices approach the profit-maximizing levels. This is not the only royalty that yields these prices as an equilibrium, however. Consider  $r = V - \frac{3}{2}t + \epsilon$ , with  $\epsilon$  small. The prices in (15) violate (1), so they do not form an equilibrium. However,  $P_H = P_I = V - \frac{1}{2}t$  is an equilibrium. To see this, note (from the reaction functions) that if it were possible for either firm to respond to the other's price of  $V - \frac{1}{2}t$  by setting its own price higher, without violating (1), then it would do so. However, pricing higher does violate this constraint, and would create local monopolies with less-than-full market coverage. We proceed to identify all per-unit royalties such that neither firm, as local monopolist, would raise its price.

For the patentee, since the market would not be fully covered, marginal changes in its price would not affect royalty revenue, so we have  $MR_H = \frac{V - 2P_H}{t}$ , which is clearly negative at  $P_H = V - \frac{1}{2}t$ . Thus, the patentee (as a local monopolist) would not raise its price. For the imitator, marginal changes in its price do affect royalty payments, and we have  $MR_I = \frac{V - 2P_I + r}{t}$ . This is non-positive at  $P_I = V - \frac{1}{2}t$  as long as  $r \leq V - t$ . Thus,  $P_H = P_I = V - \frac{1}{2}t$  is an equilibrium for  $r \in [V - \frac{3}{2}t, V - t]$ . For  $r > V - t$ , the imitator prefers to serve a smaller market to reduce royalty payments. Therefore, the bargain is efficient only if it yields a

royalty  $r \in [V - \frac{3}{2}t, V - t] \equiv \mathcal{R}^*$ . **QED**

**Proof of Proposition 1.** If bargaining breaks down and the firms compete in the shadow of the reasonable royalty regime, the firms have the following expected profit functions:

$$\begin{aligned}\pi_H^{RR} &= \frac{P_H}{2} + \frac{P_H P_I - P_H^2}{2t} + \gamma r^* \left( \frac{1}{2} + \frac{P_H - P_I}{2t} \right) + \gamma F^* \\ \pi_I^{RR} &= \frac{P_I}{2} + \frac{P_H P_I - P_I^2}{2t} - \gamma r^* \left( \frac{1}{2} + \frac{P_H - P_I}{2t} \right) - \gamma F^*,\end{aligned}\tag{16}$$

where we restrict attention to  $r^* \in \mathcal{R}^*$ . Conditional on (1) holding, taking first-order conditions and solving this system yields:

$$\begin{aligned}P_H^* &= t + \gamma r^* & D_H(P_H, P_I) &= \frac{1}{2} \\ P_I^* &= t + \gamma r^* & D_I(P_H, P_I) &= \frac{1}{2}\end{aligned}\tag{17}$$

This equilibrium therefore holds whenever  $\gamma r^* \leq V - \frac{3}{2}t$ . Because the market is split evenly, static welfare is maximized.

Now consider  $\gamma r^* \in \left( V - \frac{3}{2}t, V - t \right]$ . Similar to the proof of Lemma 1, prices following the solution to the first-order conditions are not equilibria because (1) is violated. We now analyze best responses (Figure 2 is an extremely helpful guide). When (1) holds, the optimal responses are according to the reaction functions in (14), with  $\gamma r^*$  substituted for  $r$  — for this proof, call this condition (14\*). When (1) does not hold, there are two possible best responses: (A) set price  $\underline{P}_i$ , which is such that the (local monopolist)  $MR_i = 0$ ; and (B) lower price so that (1) holds with equality — the market is fully covered and the pivotal buyer gets zero net surplus.

It is clear that, in response to an uncompetitive (i.e. extremely high) price by the other firm, either firm will choose option (A). If the other firm's price is competitive, but still high enough that responding according to (14\*) would violate (1), then, following the analysis of marginal revenues in the Proof of Lemma 1, the optimal response is (B), to set the highest price such that the constraint holds. This consists of pricing along the constraint itself. Hence, each firm's reaction function includes a section of that constraint. This section runs between where the function in (14\*) crosses the constraint and where the other firm's price becomes uncompetitive. Above this uncompetitive price, the reaction function is constant at  $\underline{P}_i$ .

For  $\gamma r^* \in \left( V - \frac{3}{2}t, V - t \right]$ , the reaction functions do indeed share a section of the constraint. Therefore, all of the equilibria in this range lie on the constraint,  $P_H + P_I = 2V - t$ , satisfying *i*. Condition *ii* follows directly from  $MR_H = \frac{V - 2P_H}{t} \leq 0$ , while *iii* follows directly

from  $MR_I = \frac{V-2P_I+\gamma r^*}{t} \leq 0$ . **QED**

**Proof of Proposition 2.** The hypothetical bargain imposes  $\gamma = 1$  if bargaining were to break down. From the Proof of Proposition 1, it is clear that there are multiple equilibria if  $\gamma r^* > V - \frac{3}{2}t$ . Hence, if bargaining in the hypothetical bargain were to break down, then for  $r^* > V - \frac{3}{2}t$ , there are multiple equilibria. Hence,  $r^U$  is the only per-unit royalty in  $\mathcal{R}^*$  such that the threat-point equilibrium is unique. **QED**

**Proof of Proposition 3.** This follows immediately from the discussion in the text. **QED**

**Proof of Proposition 4.** From the text, we have  $F = \gamma F^* + (1 - \gamma)(\beta - 1)(V - \frac{3}{2}t)$ . Let  $F^* \geq 0$  and  $\gamma < 1$ . Subtracting  $F$  from  $F^*$  yields

$$\begin{aligned} F^* - F &= F^* - \gamma F^* - (1 - \gamma)(\beta - 1)(V - \frac{3}{2}t) \\ &= (1 - \gamma) \left[ F^* + (1 - \beta)(V - \frac{3}{2}t) \right] \end{aligned}$$

Clearly, this is positive if  $F^* > 0$  and  $\gamma < 1$ .

Setting the previous equation equal to zero reveals

$$F^* = (\beta - 1)(V - \frac{3}{2}t),$$

which is clearly non-positive because  $\beta \leq 1$ . **QED**

**Proof of Proposition 5.** Under the lost profits regime, if (1) holds, then first-order conditions yield the following reaction functions:

$$\begin{aligned} P_H(P_I) &= \frac{P_I+t}{2} \\ P_I(P_H) &= \frac{(1+\gamma)P_H+t}{2}. \end{aligned}$$

Solving these two equations yields the following equilibrium:

$$\begin{aligned} P_H^* &= \frac{3t}{3-\gamma} & D_H(P_H^*, P_I^*) &= \frac{3}{6-2\gamma} \\ P_I^* &= \frac{(3+\gamma)t}{3-\gamma} & D_I(P_H^*, P_I^*) &= \frac{3-2\gamma}{6-2\gamma}. \end{aligned}$$

Given the assumption  $V \geq \frac{9t}{2(3-\gamma)}$ , these prices satisfy (1).

Thus, this equilibrium is unique by the arguments in the Proof of Proposition 1 if lost profits are indeed nonnegative in equilibrium. For  $V < 2t$ , it must be true that

$$\frac{V^2}{4t} \geq \frac{9t}{2(3-\gamma)^2}.$$

Rearranging terms, we have  $VV \geq \left(\frac{4t}{3-\gamma}\right)\left(\frac{9t}{2(3-\gamma)}\right)$ , which holds whenever  $V \geq \frac{9t}{2(3-\gamma)}$ . Since  $\frac{V^2}{4t} \geq V - t$  for all  $V$ , the condition holds for  $V \geq 2t$ .

Passive infringement is possible only if the patentee-monopolist covers less than the full market, and charges price  $\frac{V}{2}$ . To passively infringe, the imitator can charge a price no lower than  $\frac{3V}{2} - t$ . The partial derivative of profit with respect to own price,

$$\frac{\partial \pi_I^{LP}}{\partial P_I} = \frac{P_H - 2P_I + t + \gamma P_H}{2t},$$

evaluated at  $P_H = P_M = \frac{V}{2}$ , is negative if  $\frac{1}{2} + \frac{V(1+\gamma)}{4t} - \frac{3V-2t}{2t} < 0$ , or  $V > \frac{6t}{5-\gamma}$ . Given the assumption that  $V \geq \frac{9t}{2(3-\gamma)}$ , this holds, so passive infringement is not an equilibrium. **QED**

**Proof of Corollary 1.** We first show that if  $V < 2t$ , then infringement is never deterred, that is,

$$\frac{9t + 6\gamma t - 2\gamma^2 t}{2(3-\gamma)^2} - \gamma \left(\frac{V^2}{4t}\right) < 0$$

is impossible. We prove it by contradiction, so assume the above holds. Then it must be the case that  $\frac{9+6\gamma-2\gamma^2}{2\gamma(3-\gamma)^2} < \left(\frac{V}{2t}\right)^2$ . Since  $V < 2t$ , this implies that  $\frac{9+6\gamma-2\gamma^2}{2\gamma(3-\gamma)^2} < 1$ . A bit of algebra shows that this is equivalent to  $12\gamma - 10\gamma^2 + 2\gamma^3 > 9$ , which does not hold for any  $\gamma \in [0, 1]$ . Hence, the imitator will stay out of the market only if  $\pi_M = V - t$ .

Thus, infringement is deterred if and only if  $\frac{9t+6\gamma t-2\gamma^2 t}{2(3-\gamma)^2} < \gamma(V-t)$ . This condition is equivalent to  $V > \frac{9t+24\gamma t-14\gamma^2 t+2\gamma^3 t}{2\gamma(3-\gamma)^2} \equiv V^{NE}(\gamma)$ . Clearly,  $V^{NE}(\gamma) > 2t$  is finite for any  $\gamma > 0$ .

**QED**

**Proof of Proposition 6.** This follows immediately from the fact that, except for the constant term  $\gamma\pi_M$ , the profit functions for the patentee and imitator are reversed from the lost profits case. Hence, the equilibrium prices and market shares are reversed. **QED**

**Proof of Proposition 7.** Assume  $F^* = F^{Max} = \frac{t}{2}$  and  $V \geq \frac{9t}{2(3-\gamma)}$ . By the proofs of Propositions 5 and 6, the second assumption guarantees a unique equilibrium for the

lost profits and unjust enrichment regimes. The assumption  $r^* = r^U = V - \frac{3}{2}t$  guarantees that the reasonable royalty regime has a unique equilibrium. Consider first the incentives to innovate under the unjust enrichment regime compared to the other two regimes (the incentives under these assumptions are summarized in Table 1). The incentives to innovate will be greater under the reasonable royalty regime if  $\gamma(V - \frac{1}{2}t) > \frac{15\gamma t - 2\gamma^2 t}{2(3-\gamma)^2}$ . This implies

$$V > \frac{24t - 8\gamma t + \gamma^2 t}{2(3-\gamma)^2},$$

which holds for any  $\gamma$  if  $V \geq \frac{9t}{2(3-\gamma)}$ . Therefore,  $\Delta_{\Pi}^{RR} > \Delta_{\Pi}^{UR}$ . Thus, the unjust enrichment regime fails to generate the best incentives to innovate.

Now consider the incentives to innovate under lost profits compared to reasonable royalties. Assume first that infringement is not deterred under lost profits. Then  $\Delta_{\Pi}^{LP} > \Delta_{\Pi}^{RR}$  implies

$$\begin{aligned} \frac{2\gamma^2 t - 15\gamma t}{2(3-\gamma)^2} + 2\gamma(\pi_M) &> \gamma V - \frac{\gamma t}{2} \\ \Rightarrow 4(\pi_M) - 2V + \frac{-6t - 4\gamma t + \gamma^2 t}{(3-\gamma)^2} &> 0 \end{aligned}$$

Note that  $\frac{-6t - 4\gamma t + \gamma^2 t}{(3-\gamma)^2} < 0$  for all  $\gamma$ . Then, it is necessary that  $4(\pi_M) - 2V$  be positive in order that  $\Delta_{\Pi}^{LP} > \Delta_{\Pi}^{RR}$ . This cannot be true if  $V \leq 2t$ , because  $\pi_M = \frac{V^2}{4t}$  in that case. Hence, consider  $V > 2t$  and  $\pi_M = V - t$ . Continuing from above, and substituting in this condition, we get  $\Delta_{\Pi}^{LP} > \Delta_{\Pi}^{RR}$  if

$$V > \frac{3t}{2} + \frac{15t - 2\gamma t}{2(3-\gamma)^2}.$$

Assume now that entry is deterred under the lost profits regime. In this case,  $\Delta_{\Pi}^{LP} > \Delta_{\Pi}^{RR}$  if:

$$V > \frac{(2-\gamma)t}{2(1-\gamma)}.$$

Therefore, if either case holds, we have  $\Delta_{\Pi}^{LP} > \Delta_{\Pi}^{RR} > \Delta_{\Pi}^{UR}$ . Notice that as  $\gamma \rightarrow 1$ , the right-hand side of the conditional above (for when entry is precluded) approaches  $\infty$ . Thus, the only time when there will not exist a value of  $V$  such that this conditional is met is when  $\gamma = 1$ . In this case, the reasonable royalty regime generates the highest incentives to innovate for any  $V$ . **QED**

## References

- Anton, James J., and Dennis A. Yao. "Finding 'Lost' Profits: An Equilibrium Analysis of Patent Infringement Damages." *Journal of Law, Economics and Organization* forthcoming 27 (2007).
- Ayres, Ian and Paul Klemperer. "Limiting Patentee's Market Power Without Reducing Innovation Incentives: The Perverse Benefits of Uncertainty and Non-Injunctive Remedies," *Michigan Law Review* 97 (1999): 985-1033.
- Blair, R.D. and T.F. Cotter. "An Economic Analysis of Damages Rules in Intellectual Property Law," *William and Mary Law Review* 39 (1998): 1585-1694.
- Choi, Jay Pil. "Patent Litigation as an Information-Transmission Mechanism." *American Economic Review* 88 (1998): 1249-1263.
- Choi, Jay Pil. "How Reasonable is the 'Reasonable' Royalty Rate? Damage Rules and Probabilistic Intellectual Property Rights," Michigan State Working Paper (February 2006).
- Coolley, Ronald B. "Overview and Statistical Study of the Law on Patent Damages." *Journal of the Patent and Trademark Office Society* 75 (1993): 515-537.
- Crampes, Claude and Corinne Langinier. "Litigation and Settlement in Patent Infringement Cases," *RAND Journal of Economics* 33 (2002): 258-74.
- Heath, C., J. Henkel and M. Reitzig. "Who Really Profits from Patent Infringements? Innovation Incentives and Disincentives from Patent Indemnification Rules." Working Paper 2002-18, Copenhagen Business School (2002).
- Hotelling, Harold. "Stability in Competition," *Economic Journal* 39 (1929): 41-57.
- Kamien, Morton I. "Patent Licensing," R.J. Aumann and S. Hart eds. *Handbook of Game Theory 1* (1992).
- Kamien, Morton I. and Yair Tauman. "Fees Versus Royalties and the Private Value of a Patent," *Quarterly Journal of Economics* 101 (1986): 471-91.

- Katz, Michael L. and C. Shapiro. "How to License Intangible Property," *Quarterly Journal of Economics* 101 (1986): 567-89.
- Klemperer, Paul. "Optimal Patent Length and Breadth," *RAND Journal of Economics* 21 (1990): 106-12.
- Marjit, Sugata. "On a Non-Cooperative Theory of Technology Transfer," *Economics Letters* 33 (1990): 293-98.
- Mas-Colell, Andreu, Michael D. Whinston and Jerry R. Green. *Microeconomic Theory*. Oxford: Oxford University Press (1995).
- Merges, Robert and Richard R. Nelson. "On the Complex Economics of Patent Scope," *Columbia Law Review* 90 (1990): 839-916.
- Meurer, Mike. "The Settlement of Patent Litigation," *RAND Journal of Economics* 20 (1989): 77-89.
- Munson, Daniel C. "The Equitable Profits and Reasonable Royalty Remedies—An Economic Connection," *Journal of the Patent and Trademark Society* 76 (1994): 887-902.
- Poddar, Sougata and Uday Bhanu Sinha. "On Patent Licensing in Spatial Competition," *Economic Record* 80 (2004): 208-18.
- Reinganum, Jennifer F. "The Timing of Innovation: Research, Development and Diffusion," in: R. Willig and R. Schmalensee. eds., *Handbook of Industrial Organization*. Amsterdam: North-Holland (1989).
- Reitzig, Markus G., Henkel, Joachim and Heath, Christopher, "On Sharks, Trolls, and Other Patent Animals - 'Being Infringed' as a Normatively Induced Innovation Exploitation Strategy," SSRN Working Paper 885914 (2006).
- Schankerman, Mark and Suzanne Scotchmer. "Damages and Injunctions in Protecting Intellectual Property." *RAND Journal of Economics* 32 (2001): 199-220.
- Schankerman, Mark and Suzanne Scotchmer. "Still Looking for Lost Profits: The Case of Horizontal Competition," U. California-Berkeley Working Paper (2005).
- Werden, Gregory J., Luke M. Froeb and Lucian Wayne Beavers. "Economic Analysis of

Lost Profits from Patent Infringement With and Without Noninfringing Substitutes.”  
*AIPLA Quarterly Journal* 27 (1999): 305-333.

Werden, Gregory J., Luke M. Froeb and James Langenfeld. “Lost Profits from Patent Infringement: The Simulation Approach.” *International Journal of the Economics of Business* 7 (2000): 213-227.